# Nonlinear Control of a Airbreathing Hypersonic Vehicle: A Backstepping Approach

**François Poulain** * **Hélène Piet-Lahanier** ** **Laurent Serre** ***

* *ONERA-DPRS, BP72 - FR-92322 Châtillon Cedex (email:* `Francois.Poulain@onera.fr`*).*
** *ONERA-DPRS (email:* `Helene.Piet-Lahanier@onera.fr`*).*
*** *ONERA-DPRS (email:* `Laurent.Serre@onera.fr`*).*

**Abstract:** Nowadays, airbreathing vehicle's control represents a very motivating challenge, as it includes the respect of physical constraints, hard nonlinearities in thrust operation, and also a non minimum phase behavior due to elevators size. In this paper we describe a method for designing simultaneously a nonlinear guidance and control laws relying on Lyapunov functions theory. A way to handle non minimum phase dynamics is proposed, which does not require canard configuration. The control strategy aims at stabilising a dissipative phugoid motion and then back-step the wished lift behavior. A simulation case is presented, based on a realistic model of such a vehicle.

*Keywords:* Nonlinear control, Flight control, Lyapunov function, Hypersonic waveriders.

## 1. NOMENCLATURE

| Symbol | Designation | Units |
|---|---|---|
| $C_D$ | Reduced drag coefficient | |
| $\bar{C}_L$ | Reduced lift coefficient | |
| $C_M$ | Momentum coefficient | |
| $d$ | Drag factor | |
| $f$ | Lift-to-drag ratio | |
| $f_s$ | Stoichiometric factor | |
| $g_0$ | Gravity | $\mathrm{m.s^{-2}}$ |
| $h$ | Altitude | km |
| $I_{\mathrm{sp}}$ | Specific impulse | s |
| $I_{\mathrm{sp}}^V$ | Specific impulse w.r.t. $V$ | |
| $J$ | Moment of inertia | $\mathrm{kg.m^2}$ |
| $l$ | Lift factor | |
| $l_{\mathrm{ref}}$ | Reference length | m |
| $m$ | Body mass | kg |
| $\mathcal{M}$ | Mach number | |
| $q$ | Pitch rate | $\mathrm{rad.s^{-1}}$ |
| $Q_c$ | Fuel flow | $\mathrm{kg.s^{-2}}$ |
| $S_{\mathrm{ref}}$ | Wing area | $\mathrm{m^2}$ |
| $t$ | Thrust factor | $\mathrm{N.kg^{-1}}$ |
| $T$ | Thrust | N |
| $V$ | Velocity | $\mathrm{m.s^{-1}}$ |
| $\alpha$ | Angle of attack (AoA) | rad |
| $\gamma$ | Flight path angle | rad |
| $\delta$ | Angle of elevators | rad |
| $\varepsilon$ | Air capturing area | $\mathrm{m^2}$ |
| $\eta$ | Thrust ratio | |
| $\rho$ | Air density | $\mathrm{kg.m^3}$ |
| $\Phi_i$ | Air to fuel ratio | |

## 2. INTRODUCTION

The wide interest of airbreathing hypersonic propulsion had been established for a long time in various applications. As a matter of fact, the availability of air-picked oxydizer during atmospheric phase of launcher's flight allows to increase dramatically specific impulse of the propulsion system. In these conditions, payload capacity is significantly better compared to a classical rocket propulsion. Research about hypersonic airbreathing technologies has been studied for a long time in Onera, currently during the Lea demonstrator project described in Falempin and Serre (2008).

However, this positive picture has to be balanced with strong technological challenges and the difficulty to reproduce operating conditions in ground test facilities. In a control perspective, these vehicles are subject to specificities which imply the need to investigate dedicated control methods (see Doman et al. (2006) for an overview). Among these specificities, we can outline: (i) strong coupling between thrust and attitude; (ii) high modelling uncertainties due to unknown aero-thermo-dynamic behavior (iii) large operation range.

We distinguish three main approaches in literature. The first, described in Chavez and Schmidt (1992, 1993); Schmidt and Velapoldi (1999), was relied to classical control design in frequency domain. These works emphasized the requirement for such a control design to take into account the coupling between the engine and airframe which leads to nonlinear behavior.

The second approach was initiated by Bolender and Doman (2007) which has brought a knowledge model for control design, based on physical modelling. This model put the emphasis on an unstable inverse dynamic behavior, as well as slightly damped high frequency dynamics. From this, some linear MIMO control design have been investigated (see Parker et al. (2007); Sigthorsson et al. (2008) and references inside for an overview). However, these works involve linear design while operation range of hypersonic vehicles is typically large (particularly with respect to Mach). As a consequence, to design a complete control law for a vehicle operating in a multi-dimensional state range, interpolation techniques have to be used, and leads to control structure complex to analyse, which can introduce local instability of closed loop behavior. In order to overcome these limitations, Fiorentini et al. (2009) recently proposed a Lyapunov based adaptive control addressing structural flexibility effects, where a canard deflector is used to decouple lift from elevator commands, then avoiding instability of the inverse dynamics.

The third approach follows the path charted by Wang and Stengel (2000). Using a vehicle model similar to an air plane, control synthesis has been done using nonlinear dynamic inversion. However, if on one hand this technique is quite generic and theoretically leads to global properties, on the other it is known to lack of robustness w.r.t. modelling uncertainties. Moreover, it needs huge amount of speculative *a priori* information about the process to be controlled, and leads to quite complex control laws. It is quite difficult to embed and to study robustness of such control laws. The works of Xu and Ioannou (2004) and Fidan et al. (2003) improved these limitations by using high gain sliding mode techniques. However high gain techniques are also known to introduce high frequency excitation of actuators, which render them difficult to implement. Furthermore, these works did not address the issue of inverse instable dynamics highlighted in Bolender and Doman (2007), because dynamic inversion as well as robust control are inadequate in this context.

These different approaches reveal a convergence of views about difficulties to overcome in order to design control of airbreathing hypersonic vehicles. Among them: (i) handling the inverse instable dynamic; (ii) the determination of a global (or semi-global) solution verifying robustness properties; (iii) the compliance with physical limits with respect to vehicle integrity. Nevertheless none of the above approaches handles simultaneously these three important points.

This paper aims at proposing a control design which takes care of the three points described above. It consist in the definition of a dynamical model of an airbreathing hypersonic vehicle, as well as a nonlinear control structure with global properties. To this end, we investigate the Lyapunov synthesis of a gradient control to stabilize a velocity vector given by speed and path angle. The main interests of gradient control are its robustness (especially if it does not imply to embed unknown information in the controller), possible optimality properties, and its ability to trade with saturations. By doing so, we obtain an attitude reference which acts as an intermediate control law. We then back-step this reference in order to get the elevators control law.

The paper is organized as follow. We firstly describe in Section 3 the different assumptions made on the modelling of the aerodynamics and thrust. Our model structure is designed from semi-empiric methods. The resulting model is close to what is described in Parker et al. (2007). We thus describe in Section 4 the model structure kept for control design. We next design in Section 5 a control law stabilizing a constant reference trajectory given in terms of flight speed and path angle. We aim at designing in Section 6 a closed-loop guidance and control law that will stabilize the vehicle along this trajectory. Finally, a simulation case is presented in Section 7 to illustrate the behavior obtained by the approach.

## 3. PHYSICAL MODEL OF THE VEHICLE

### 3.1 Aerodynamic forces

Aerodynamic forces are usually modelled as a product of dynamic pressure —which depends of relative air velocity and air density—, and a coefficient term reflecting wetted surface of the wing. This coefficient depends on wing geometry and relative air flow direction:

$$F_\star = \frac{1}{2}\rho V^2 S_{\mathrm{ref}} C_\star, \tag{1}$$

where $C_\star$ is an *aerodynamic coefficient*. It varies with angle of attack, angle of elevators and Mach number; as Figure 1 illustrates, it is well known in the hypersonic domain that the $C_\star$ coefficients decrease w.r.t. Mach number. Here we rather use a "scaled" aerodynamic force model

$$F_\star = \frac{1}{2}\rho(h) S_{\mathrm{ref}} V^2 S(V) C_\star(\alpha, \delta), \tag{2}$$

where $S(V)$ is a similitude factor adapted to hypersonic domain. We use the empiric function learned from data set $S(V) = (0.3 + 0.13\mathcal{M})^{-1}$. From this, in the following we aggregates the speed influence on aerodynamics force by defining $\Lambda(V) = V^2 S(V)$. This define a lift-to-drag ratio which is independent with respect to the Mach.

We finally modelled our forces coefficients as a linearly parameterized function in $(\alpha, \delta)$ using data obtained from semi-empiric methods (see Figure 1 and 2):

$$C_L = \mathbf{L}_L \,(1 \;\, \alpha \;\, \delta)^\top, \tag{3}$$

$$C_D = \mathbf{L}_D \,(1 \;\, \alpha \;\, \delta)^\top + (\alpha \;\, \delta)\, \mathbf{Q}_D \,(\alpha \;\, \delta)^\top. \tag{4}$$

### 3.2 Aerodynamic moment

Aerodynamic moment is usually modelled as

$$M_\star = \frac{1}{2}\rho V^2 S_{\mathrm{ref}} l_{\mathrm{ref}} C_\star. \tag{5}$$

As far as is concerned aerodynamic pitch coefficient, it is roughly linear as it can be seen on Figure 2. Anyway, for nonlinear control design we can consider any model affine in input:

$$C_{\scriptscriptstyle M} = C_{\scriptscriptstyle M0}(\alpha, \mathcal{M}, q) + C_{\scriptscriptstyle M\delta}(\alpha, \mathcal{M}, q)\delta \tag{6}$$

where $q$ acts mainly as a damping effect.

### 3.3 Scramjet thrust

Scramjet propulsion is a very complex phenomenon which we describe using a specific impulse function depending on Mach number and air-fuel ratio:

$$T = g_0 Q_c I_{\mathrm{sp}}(\mathcal{M}, \Phi_i). \tag{7}$$

Figure 3 illustrates the input/output behavior of such a knowledge model. To construct a behavior model, it appears that specific impulse can itself be split as a product of two terms:

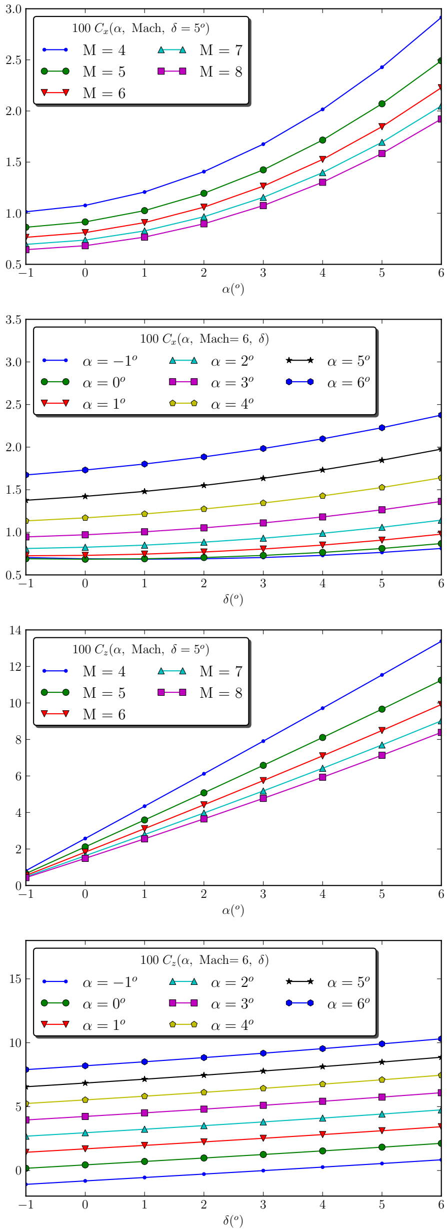


Fig. 1. Aerodynamic forces coefficients with respect to angle of attack, angle of elevators, and Mach number.

- the dependency on Mach —relying on efficiency— monotonic and decreasing in all hypersonic domain, which can be assimilated to a dependency in $V$;
- the dependency on air-fuel ratio, a unimodal function passing through a maximum near $\Phi_i = 0.7$.

This leads to consider $I_{\text{sp}}(\mathcal{M}, \Phi_i) = I_{\text{sp}}^{\Phi_i}(\Phi_i) I_{\text{sp}}^V(V)$. But as

$$Q_c = \Phi_i f_s Q_A = \Phi_i f_s \rho V \bar{\varepsilon}(\alpha, \mathcal{M}), \tag{8}$$

where $\bar{\varepsilon}$ is air-captation area, we then can rewrite (7) as

$$T = g_0 f_s \rho \bar{\varepsilon}(\alpha, \mathcal{M}) \Phi_i I_{\text{sp}}^{\Phi_i}(\Phi_i) V I_{\text{sp}}^V(V). \tag{9}$$

Moreover, since the Scramjet is usually designed to keep the term $\rho \bar{\varepsilon}(\alpha_0, \mathcal{M}) V$ constant with respect to Mach, we thus can model thrust as

$$T = m \varepsilon(\alpha) I_{\text{sp}}^V(V) \eta, \qquad \eta \in [0; 1] \tag{10}$$

where $\eta \propto \Phi_i I_{\text{sp}}^{\Phi_i}(\Phi_i)$ is a bounded propulsion characteristic which is assumed to be locally invertible. Our scaled air-

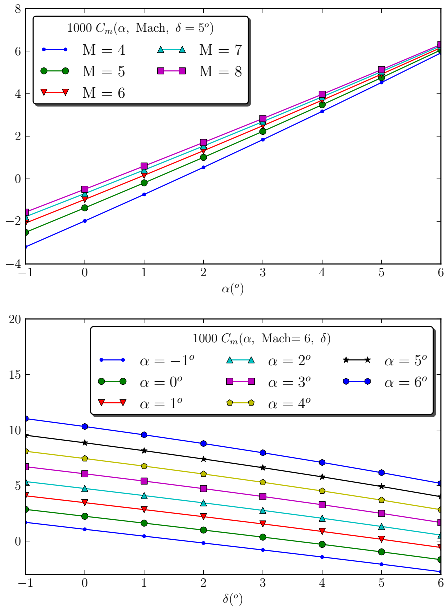


Fig. 2. Aerodynamic moment coefficient with respect to angle of attack, angle of elevators, and Mach number.

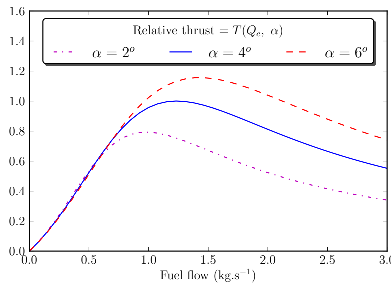


Fig. 3. Thrust force variation with respect to angle of attack and fuel flow rate.

captation model is considered as a locally affine function of angle of attack

$$\varepsilon(\alpha) = \max(\varepsilon_0, \min(\varepsilon_{00} + \varepsilon_{0\alpha}\alpha, \varepsilon_{10} + \varepsilon_{1\alpha}\alpha)), \tag{11}$$

and thrust efficiency w.r.t. Mach as

$$I_{\text{sp}}^V(V) = (i_0 + i_{\mathcal{M}} \mathcal{M})^{-1}, \tag{12}$$

where $i_0$ and $i_{\mathcal{M}}$ are assumed constants.

*Remark 1.* We assume in this paper that $\eta$ can be a controlled parameter. This requires knowing the air flow rate passing through the propulsion and having good description of the characteristic $\Phi_i I_{\text{sp}}^{\Phi_i}(\Phi_i)$. The latter is possible since extensive testing of propulsion is realized during vehicle's conception. Using our propulsion modelling, this characteristic is a smooth unimodal function vanishing at origin with a maximum which actually limits the propulsion operating range.

## 4. DESCRIPTION OF THE CONTROLLED MODEL

### *4.1 Rigid body mechanics*

We restrict ourselves to the vertical plane and assume that the mass does not vary during flight. In this context, by noting $\mathbf{R}_{\mathrm{A}\to 0}$ the orthogonal matrix transforming a velocity related frame to the inertial frame, we can write

$$\dot{\widehat{\mathbf{R}_{\mathrm{A}\to 0}\mathbf{V}}} = \mathbf{R}_{\mathrm{A}\to 0}\dot{\mathbf{V}} + \frac{\partial \mathbf{R}_{\mathrm{A}\to 0}}{\partial \gamma}\mathbf{V}\dot{\gamma} = \mathbf{R}_{\mathrm{A}\to 0}\frac{\mathbf{F}}{m}, \qquad (13)$$

and left-multiplying (13) by $\mathbf{R}_{\mathrm{A}\to 0}^{\top}$ leads to the expression of the velocity dynamics $(V, \gamma)$.

Moreover, since the dynamics modeling is restricted to the plane $(O, \boldsymbol{x}_0, \boldsymbol{z}_0)$, the rotational dynamics is straight forward. Hence the complete modelling of the vehicle is defined in vertical plane by

$$\dot{V} = \frac{1}{m}\mathbf{F}.\boldsymbol{x}_{\mathrm{A}}, \qquad \dot{\gamma} = \frac{1}{mV}\mathbf{F}.\boldsymbol{z}_{\mathrm{A}}, \qquad (14a)$$

$$\dot{\alpha} = q - \dot{\gamma}, \qquad \dot{q} = \frac{M}{J}, \qquad (14b)$$

where $\boldsymbol{x}_{\mathrm{A}}$ and $\boldsymbol{z}_{\mathrm{A}}$ are velocity related frame axes.

### *4.2 Shaping up the plant in feedback form*

As can be seen on Figure 1, there is a strong dependency of force coefficients w.r.t. elevator angles. This technological specificity is due to the presence of huge control wings necessary to counteract the moment induced by the intrados of the vehicle. In a control perspective, such a feature is known to induce a non-minimum phase behavior; for more details, see *e.g.* Parker et al. (2007) and discussion in Menon (2001) following Wang and Stengel (2000). This specificity of waveriders concepts is not observed on most aircraft models, where the forces induced by control surface are generally neglected. In Sigthorsson et al. (2008) and Fiorentini et al. (2009), the authors use canard configuration to handle this difficulty. Another way to handle this direct force effect has been proposed (see *e.g.* Shkolnikov and Shtessel (2001)) in the context of sliding mode control, using output redefinition.

Since there exists a time scale separation between rotational dynamics and velocity dynamics, we treat this problem as a singularly perturbed hierarchical control.

### *4.3 Singularly perturbed hierarchical control*

*Theorem 2.* Let a singular perturbed and sufficiently smooth system be

$$\dot{x} = f(x, z, u), \qquad (15a)$$

$$\epsilon\dot{z} = g(x, z, u), \qquad (15b)$$

where $g(x, z, u)$ is affine in $u$ and $u = \phi_f(z, z_c)$ is a fast $x$-parameterized control law stabilising exponentially $z \to z_c$ uniformly in $x$ when $\epsilon \to 0$. Suppose that the equation $g(x, z_c, u) = 0$ have only one isolated root given by $u = \varphi(x, z_c)$ which describes the fast equilibrium manifold.

Then, there exists a positive time delay $\Delta t$ and a minimum scale separation $\epsilon^*$ such that if the perturbed system

$$\dot{x} = \bar{f}(x, z_c) = f(x, z_c, \varphi(x, z_c)) \qquad (16)$$

has its origin asymptotically stable, then the overall system (15) has its origin asymptotically stable $\forall t \in [t_0 + \Delta t; \infty[$ for a sufficiently small time scale factor $\epsilon < \epsilon^*$.

It will be shown that since the system is singularly perturbed, the slow subsystem is equivalent to its approximation given when $u$ is replaced by its value on the fast equilibrium manifold. This triangularises the overall system. This way, and using the above assumptions, we can stabilise the origin of (15) with a control law designed on the following lower triangular system

$$\dot{x} = \bar{f}(x, z), \qquad (17a)$$

$$\epsilon\dot{z} = g(x, z, u). \qquad (17b)$$

To this end, a backstepping procedure allows simplifying the nonlinear control design.

**Proof.** Since fast subsystem is affine in control, we can write

$$u = \varphi(x, z_c) + \tilde{\phi}_f(\tilde{z}), \qquad (18)$$

where $\tilde{z} = z - z_c$ is the distance to the equilibrium manifold. Thus, the boundary layer equation

$$\dot{\tilde{z}} = \tilde{g}_x(z_c, \tilde{z}, \tilde{\phi}_f(\tilde{z})) \qquad (19)$$

is exponentially stable by assumption. By application of Tikhonov's theorem, present in Kokotovic et al. (1986), we know that there exists $\epsilon^*$ and $\Delta t$ such that under assumptions the perturbed system (17) is equivalent to the original system (15) over a time delay $\Delta t$. This completes the proof.

### *4.4 Hierarchical control of hypersonic vehicle*

A critical step for the use of the preceding theorem is to solve the fast equilibrium manifold equation $g(x, z, u) = 0$. In aeronautics, this equation relies on the rotational dynamics equilibrium equation. Assuming we have a linear approximation of the aerodynamic coefficient expression, the fast equilibrium manifold is given by

$$C_M = 0 \approx C_M(\alpha_0, q_0, \mathcal{M}_0, \delta_0) + \frac{\partial C_M}{\partial \alpha}\alpha + \frac{\partial C_M}{\partial \delta}\delta, \quad (20)$$

and theorem 2 allows us to consider that angle of elevators have to stabilise fast dynamics so quickly that its value on the equilibrium manifold is a sufficiently good approximation to use it in the slow dynamics. Therefore, we can rewrite equation (3) on the equilibrium manifold as

$$\bar{C}_L = C_{L_0} + C_{L_\alpha}\alpha. \qquad (21)$$

The application to the drag coefficient lead us to consider a classical model $\bar{C}_D = C_{D0} + C_{Dl}\bar{C}_L^2$. This model have a sense in the scaled aerodynamic model, since the lift-to-drag ratio is Mach-independent.

In order to complete our model, we make the following assumptions:

*Assumption 1.* The curvature of the earth is neglected and Coriolis force is replaced by a constant offset on gravity.

*Assumption 2.* The air density $\rho(h)$ is supposed to be constant for the domain of altitude.

*Assumption 3.* Thrust $T$ is assumed to be state independent.

Then we write the $(V, \gamma)$ dynamics as

$$\dot{V} = t - d(l)\Lambda(V) - g\sin\gamma, \qquad (22a)$$

$$\dot{\gamma} = \frac{l\Lambda(V) - g\cos\gamma}{V}, \qquad (22b)$$

where $t = \frac{T}{m}$, $d(l) = \frac{1}{2m}\rho(h)S_{\mathrm{ref}}\bar{C}_D$ and $l = \frac{1}{2m}\rho(h)S_{\mathrm{ref}}\bar{C}_L$. Such a system is a variation of the Zhukovskii oscillator (see Andronov et al. (1987) and references inside).

## 5. STABILITY AND STABILISATION OF THE $(V,\gamma)$ SLOW SUBSYSTEM

### 5.1 *Existence and uniqueness of equilibrium*

Let $f$ be the lift-to-drag ratio; the speed equilibrium is given by solving the polynomial equation

$$(l\Lambda(V))^2 + \left(t - \frac{l}{f}\Lambda(V)\right)^2 - g^2 = 0. \tag{23}$$

Since $\Lambda(V)$ maps $\mathbb{R}_+ \to \mathbb{R}_+$, we are looking for real positive root of equation (23). Computation of the discriminant to find a condition for real root existence leads to the condition

$$t^2 < g^2\left(1 + \frac{1}{f^2}\right). \tag{24}$$

Moreover there exists only one positive solution if $t^2 < g^2$. Else, in the case

$$g^2 < t^2 < g^2\left(1 + \frac{1}{f^2}\right), \tag{25}$$

there exists another (low-speed and high flight path angle) equilibrium position where flight is mainly sustained by thrust. However, this equilibrium is not in the foreground of this study, since hypersonic atmospheric flight cannot be maintained with such high flight path angle.

*Assumption 4.* The hypersonic airbreathing vehicle is supposed to keep the flight path angle low, *e.g.* $\gamma$ is small.

This last assumption simplifies $(V,\gamma)$ dynamics as

$$\dot{V} = t - d(l)\Lambda(V) - g\gamma, \tag{26a}$$

$$\dot{\gamma} = \frac{l\Lambda(V) - g}{V}, \tag{26b}$$

and an approximation of the equilibrium is given by

$$\gamma_0 = \frac{t_0}{g} - \frac{1}{f_0}, \qquad V_0 = \Lambda^{-1}\left(\frac{g}{l_0}\right). \tag{27}$$

### 5.2 *Stability of the equilibrium*

*Definition 1.* ($\mathcal{C}^1$-dissipativeness). The system $\dot{x} = \zeta(x)$ is $\mathcal{C}^1$-dissipative if there exists a $\mathcal{C}^1$ Lyapunov function $W$ (*i.e.* positive definite and proper) satisfying

$$\frac{\partial W}{\partial x}(x)\zeta(x) \leq 0 \quad \forall x \in \mathbb{R}^n.$$

*Proposition 3.* (Dissipativeness of flight). Given three positive constants $l_0$, $d_0$ and $t_0 < g$, the Zhukovskii oscillator defined by (26) is $\mathcal{C}^1$-dissipative and its unique equilibrium is exponentially stable.

**Proof.** Uniqueness of equilibrium is already established, so we focus on stability. Choosing as a Lyapunov function

$$W(V,\gamma) = \int_{\Lambda^{-1}\left(\frac{g}{l_0}\right)}^{V} \frac{\Lambda(v) - \frac{g}{l_0}}{v}dv + \frac{g}{2l_0}\left(\gamma - \frac{t_0}{g} + \frac{d_0}{l_0}\right)^2, \tag{28}$$

with

$$\frac{\partial W}{\partial V} = \frac{\Lambda(V) - \frac{g}{l_0}}{V}, \quad \frac{\partial W}{\partial \gamma} = \frac{g}{l_0}\left(\gamma - \frac{t_0}{g} + \frac{d_0}{l_0}\right), \tag{29}$$

leads to

$$\dot{W}(V,\gamma) = -\frac{d_0}{V}\left(\Lambda(V) - \Lambda(V_0)\right)^2. \tag{30}$$

This proves the $\mathcal{C}^1$-dissipativeness of (26). Secondly, given a compact set $C$ included in a neighborhood of the equilibrium point, the set $\{(V,\gamma) \in C : \dot{W}(V,\gamma) = 0\}$ contains a unique invariant point $(V_0,\gamma_0)$. Therefore, using LaSalle theorem we can establish the asymptotic stability of $(V_0,\gamma_0)$.

Exponential stability can be proved by studying the equilibrium tangent's approximation: under the condition [1]

$$\frac{\partial \Lambda}{\partial V} < 4f^2\frac{\Lambda(V_0)}{V_0}, \tag{31}$$

the real part of the eigenvalues of the dynamics first order approximation is given by $-\frac{l_0}{2f}\frac{\partial \Lambda}{\partial V}$. This ends the proof.

### 5.3 *Stabilisation of the $(V,\gamma)$ subsystem*

The reference trajectory is assumed to be given in terms of a sequence of $(V_c,\gamma_c)$ to be reached. Thus we can find using (27) a couple $l_c$ and $t_c$, such that equilibrium of the system (26) coincides with $(V_c,\gamma_c)$.

From the $\mathcal{C}^1$-dissipativeness of the system, it appears that the stabilisation of the system's origin can be achieved by any control law of the form $u = \varphi\left(-L_gW(V,\gamma)\right) + u_c$, where $\varphi$ is a monotonic increasing $\mathcal{C}^0$ function such that $\forall x \in \mathbb{R}, \varphi(x)x \geq 0$. From this, a known property of such a gradient control is it robustness, since it does not require the perfect knowledge of $L_gW(V,\gamma)$ to ensure the stability of the trajectories. This also allows using bounded functions as control laws.

For example, defining $\mathrm{sat}(x,a,b) = \min(\max(x,a),b)$; a couple of control laws for $l$ and $t$ could be

$$l = \mathrm{sat}\left(-k_\gamma\frac{\partial W}{\partial \gamma}V + l_c, \underline{l}, \bar{l}\right), \tag{32}$$

$$t = \mathrm{sat}\left(-k_V\frac{\partial W}{\partial V} + t_0, \underline{t}, \bar{t}\right) + (d(l) - d(l_c))\Lambda(V), \tag{33}$$

which can be bounded if we restrict $\Lambda(V)$ to a closed set.

### 5.4 *Extension to the airbreathing case*

We no longer take into account Assumption 3. In equation (10) thrust was written as function of $\alpha$ and $V$. But from (21) a certainty equivalence exists between $\alpha$ and $l$. So we can rewrite thrust as

$$t = \varepsilon(l)I_{\mathrm{sp}}^V(V)\eta, \tag{34}$$

where $\varepsilon$ is a saturated function, locally affine in $l$, obtained by replacing $\alpha$ in (11). The control inputs are definitly $\eta$ and $l$; so the $(V,\gamma)$ subsystem has to be rewritten as

$$\dot{V} = \varepsilon(l)I_{\mathrm{sp}}^V(V)\eta - d(l)\Lambda(V) - g\gamma, \tag{35a}$$

$$\dot{\gamma} = \frac{l\Lambda(V) - g}{V}, \tag{35b}$$

for which exists an equilibrium given by

$$V_0 = \Lambda^{-1}\left(\frac{g}{l_0}\right), \qquad \gamma_0 = \frac{\varepsilon(l_0)I_{\mathrm{sp}}^V(V_0)\eta_0}{g} - \frac{1}{f_0}. \tag{36}$$

*Proposition 4.* ( $\mathcal{C}^1$-dissipativeness of airbreathing flight ). Given three positive constants $l_0 : \varepsilon(l_0) > 0$, $d_0$ and $\eta_0 : t(l_0,V_0) < g$, the airbreathing flight defined by (35) is $\mathcal{C}^1$-dissipative and its equilibrium, defined by (36), is asymptotically stable.

[1] Verified in practice; when $\Lambda(V) = V^2$ it reduce to $f^2 > \frac{1}{2}$.

**Proof.** Derivating Lyapunov function (28) leads to

$$\dot{W}(V,\gamma) = -\frac{d_0}{V}\left(\Lambda(V)-\Lambda(V_0)\right)^2 + \frac{\varepsilon(l_0)\eta_0}{V}\left(\Lambda(V)-\Lambda(V_0)\right)\left(I_{\rm sp}^V(V)-I_{\rm sp}^V(V_0)\right). \quad (37)$$

Since $\varepsilon(l_0)\eta_0$ is positive, and $\Lambda$ and $I_{\rm sp}^V$ are monotonic functions respectively increasing and decreasing, we conclude to the $\mathcal{C}^1$-dissipativeness of system (35). Asymptotic stability can be established similarly as in proposition 3.

Due to the bilinear input effect, controlling system (35) leads to a control allocation problem between thrust and lift effect. This is nothing but the consequence the thrust availability is subject to controlled angle of attack (AoA): low AoA may render unaccessible trajectory due to a lack of available thrust, and in other hand high AoA dramatically increases drag which affects propulsive balance. Solving such a nonlinear problem —*e.g.* by cost formulation— is a difficult problem which will not be addressed here, but this motivated us to restrict admissible values of AoA in a neighbour of equilibrium. So, in a perspective of extending control law (33) we will keep the same $l$ as in Section 5.3. By noting $l_c$ and $\eta_c$ the couple of constant controls such that equilibrium of the $(V,\gamma)$ subsystem coincides with $(V_c,\gamma_c)$, a stabilising control law for the system (35) is given by

$$l = \mathrm{sat}\left(-k_\gamma\frac{\partial W}{\partial\gamma}V + l_c, \underline{l}, \bar{l}\right), \quad (38)$$

$$\eta = \eta_c + \frac{1}{\varepsilon(l)I_{\rm sp}^V(V)}\Big\{\mathrm{sat}\left(-k_V\frac{\partial W}{\partial V}\varepsilon(l)I_{\rm sp}^V(V), \underline{t}, \bar{t}\right) + (d(l)-d(l_c))\Lambda(V) - \eta_c I_{\rm sp}^V(V_c)\big(\varepsilon(l)-\varepsilon(l_c)\big)\Big\}, \quad (39)$$

which can be bounded if we restrict $V$ to a closed set.

## 6. BACKSTEPPING OF THE LIFT CONTROL LAW

As the slow subsystem control law has been designed, backstepping is a powerful tool to find a control Lyapunov function stabilising the overall system by extending (28). We need two back steps in order to stabilize lift and pitch dynamics.

### 6.1 First step: stabilisation of $\alpha$

We extend the dynamics of the system to

$$\dot{V} = \varepsilon(l)I_{\rm sp}^V(V)\eta - d(l)\Lambda(V) - g\gamma, \quad (40a)$$

$$\dot{\gamma} = \frac{l\Lambda(V)-g}{V}, \quad (40b)$$

$$\dot{l} = C_{L_\alpha}\left(q-\dot{\gamma}\right). \quad (40c)$$

Let $\phi_\gamma(\gamma)$ be the control law defined in (38), the Lyapunov function [2]

$$W_2(V,\gamma,l) = k_\epsilon W(V,\gamma,l) + \frac{1}{2}\left(l-\phi_\gamma(\gamma)\right)^2 \quad (41)$$

is derived along the solutions as

$$\dot{W}_2(V,\gamma,l) = k_\epsilon\dot{W}(V,\gamma,\phi_\gamma(\gamma)) - C_{L_\alpha}\frac{\Lambda(V)}{V}\left(l-\phi_\gamma(\gamma)\right)^2 + \left(l-\phi_\gamma(\gamma)\right)\Big[k_\epsilon\frac{\partial W}{\partial\gamma}\frac{\Lambda(V)}{V} + C_{L_\alpha}\left(q-\frac{\phi_\gamma(\gamma)\Lambda(V)-g}{V}\right) - \dot{\widehat{\phi_\gamma(\gamma)}}\Big], \quad (42)$$

and therefore is definite negative by choosing

$$q = \mathrm{sat}(-k_l(l-\phi_\gamma(\gamma)), \underline{q}, \bar{q}) + \mathcal{Q} := \phi_l(V,\gamma,l), \quad (43)$$

where

$$\mathcal{Q} = C_{L_\alpha}^{-1}\left(\dot{\widehat{\phi_\gamma(\gamma)}} - k_\epsilon\frac{\partial W}{\partial\gamma}\frac{\Lambda(V)}{V} + C_{L_\alpha}\frac{\phi_\gamma(\gamma)\Lambda(V)-g}{V}\right).$$

### 6.2 Second step: stabilisation of $q$

As equation (43) define $q=\phi_l(V,\gamma,l)$ as an intermediate control law, then we consider

$$\dot{V} = \varepsilon(l)I_{\rm sp}^V(V)\eta - d(l)\Lambda(V) - g\gamma, \quad (44a)$$

$$\dot{\gamma} = \frac{l\Lambda(V)-g}{V}, \quad (44b)$$

$$\dot{l} = C_{L_\alpha}\left(q-\dot{\gamma}\right), \quad (44c)$$

$$\dot{q} = \frac{1}{2J}\rho V^2 S_{\rm ref}l_{\rm ref}\left[C_{M0}+C_{M\delta}\delta\right], \quad (44d)$$

and the Lyapunov function

$$W_3(V,\gamma,l,q) = W_2(V,\gamma,l,\phi_l(l)) + \frac{1}{2}(q-\phi_l(l))^2. \quad (45)$$

Its derivative along the solutions is

$$\dot{W}_3(V,\gamma,l,q) = \dot{W}_2(V,\gamma,l,\phi_l(l)) + (q-\phi_l(l))\Big[C_{L_\alpha}(l-\phi_\gamma(\gamma)) + \dot{q} - \dot{\widehat{\phi_l(l)}}\Big]. \quad (46)$$

Finally, choosing the control law

$$\delta = \mathrm{sat}(-k_q\frac{C_{M\delta}}{2J}\rho V^2 S_{\rm ref}l_{\rm ref}(q-\phi_l(l))), \underline{\delta}, \bar{\delta}) + \mathcal{D} \quad (47a)$$

$$:= \phi_q(V,\gamma,l,q) \quad (47b)$$

globally stabilizes the hypersonic vehicle's model defined by (44), where

$$\mathcal{D} \;=\; -\frac{C_{M0}}{C_{M\delta}} + \frac{2J\left(\dot{\widehat{\phi_l(l)}} - C_{L_\alpha}(l-\phi_\gamma(\gamma))\right)}{\rho V^2 S_{\rm ref}l_{\rm ref}C_{M\delta}}. \quad (48)$$

Since $q$ acts as a damping term, it contributes to stabilization; then we consider $q=0$ in $C_{M0}$ and $C_{M\delta}$.

## 7. SIMULATION RESULTS

The simulation results of the control law $\phi_q(V,\gamma,l,q)$ (with $\gamma_c$ chosen in order to stabilize altitude) are presented on Figure 4. The simulation have been made on a small scale demonstrator ($M\;=5000$ kg, $S_{\rm ref}\;=7$ m$^2$, $l_{\rm ref}\;=6.9$ m). We used for simulation a fully nonlinearly parameterised aerodynamics and an altitude-dependent perturbation wind. The model parameters used to implement the control were vitiated by variation of $\pm 20\%$. This acts on aerodynamic coefficients, as well as propulsion parameters (efficiency, air captation area). Moreover, even if the thrust control is given in terms of $\eta$, the simulated control embedded is function of fuel flow rate $Q_c$. It has been computed using a vitiated knowledge of the amount of air in the airbreathing propulsion, and the characteristic

[2] The $k_\epsilon$ coefficient is needed because of time scale separation.

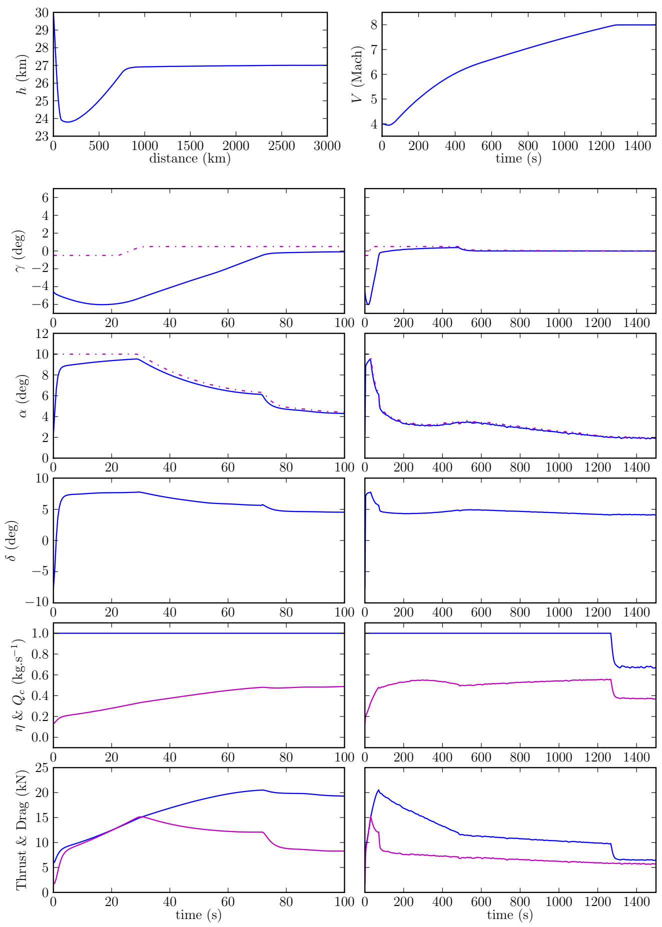


Fig. 4. Stabilisation of a representative nonlinear vehicle using the control law $\phi_q(V, \gamma, l, q)$.

$\eta(\Phi_i)$ has been approximated to the identity in the control implementation, in order to calculate the controlled fuel flow rate. All these modelling vitiations between the simulated model and the behavior model used to design control are here to illustrate the robustness of our approach.

The available tuning parameters are $(k_\gamma, k_V, k_\epsilon, k_l, k_q)$ and the saturations bounds. The latter have been chosen to keep state in classical values, but no special gain synthesis have been done. In order to illustrate the limits of the controlled airbreathing hypersonic vehicle (HSV), initialisation has been set far from equilibrium. As it can be seen, the influence of propulsion on rotational dynamics is clearly covered by aerodynamic forces, and robustness to parameters uncertainties is satisfied. The flight simulation results have been cut in two phases.

The first phase concerns the stabilisation of the path angle, while the vehicle has been initialized in tough conditions. Since the control law has been shaped in order to saturate the AoA of the vehicle, first it is saturated by an angle limit (*e.g.* to guarantee air inlet properties), and next, after altitude decreases, AoA is saturated by vertical load factor (*e.g.* to preserve body's structure). This allows to guaranty integrity of the vehicle, even in difficulty situations. Next, the vehicle continues its speed increase to reach a Mach 8 reference. The main part of propulsion available is then used during acceleration of the HSV.

The control allocation problem discussed in Section 5.4 appears slightly these two phases. In high AoA situation, the drag appears to grow quicker than the propulsion availability, which imply the reduction of the propulsive balance. Also, during the cruise, the mass of the vehicle is decreasing. Since the altitude is constant and the speed vary slowly, this imply the decreasing of the angle of attack. Consequently, the decreasing of the AoA imply a thrust attenuation.

## 8. CONCLUSION

The global nonlinear control law which has been addressed here is an attempt to assign a Lyapunov based flight nonlinear control law, allowing to trade with nonlinearities like state saturations. The assumptions made here are not very restrictive. Extension of such a control law can be performed on any flying vehicle with little small path angle and rigid body.

There is many perspectives opened by such a nonlinear control law. To this end, we have to extend our study to altitude's dynamics, and to study control design on a vehicle with 6 degrees of liberty. This can be done using similar design if we restrict us to bank-to-turn flight, knowing that this kind of vehicle avoid the ability of side-slipping flight. Others perspectives are to trade with the control allocation problem. We also need to robustify this control law to model uncertainties, using *e.g.* nonlinear integrators. Thirdly, we have to find guarantees of boundedness of $\eta$ without obtaining suboptimal gains on $k_V$. Finally we need to find robust control laws guaranteeing stability under saturated angle of elevators $\delta$.


## ACKNOWLEDGEMENTS

The authors are very grateful to Dr. Laurent Praly for his numerous advices about nonlinear control techniques.


## REFERENCES


Andronov, A.A., Vitt, A.A., and Khaikin, S.E. (1987). *Theory of Oscillators*. Dover.

Bolender, M.A. and Doman, D.B. (2007). Nonlinear longitudinal dynamical model of an air-breathing hypersonic vehicle. *AIAA Journal of Spacecraft and Rockets*, 44(2), 374–387.

Chavez, F.R. and Schmidt, D.K. (1992). An integrated analytical aeropropulsive/aeroelastic model for the dynamic analysis of hypersonic vehicles. In *AIAA Atmospheric Flight Mechanics Conference*. Hilton Head Island.

Chavez, F.R. and Schmidt, D.K. (1993). Dynamics of hypersonic flight vehicles exhibiting significant aeroelastic and aeropropulsive interactions. In *AIAA Guidance, Navigation and Control Conference*. Monterey, CA.

Doman, D., Oppenheimer, M., and Bolender, M. (2006). Progress in guidance and control research for space access and hypersonic vehicles. In *Decision and Control, 2006 45th IEEE Conference on*, 6837 –6851.

Falempin, F. and Serre, L. (2008). French R&T effort for high-speed airbreathing propulsion Status in 2008. In *59th International Astronautical Congress (IAC-08-C4.5.5)*. Glasgow, Scotland.

Fidan, B., Mirmirani, M., and Ioannou, P.A. (2003). Flight dynamics and control of air-breathing hypersonic vehicles: Review and new directions. In *12th AIAA International Space Planes and Hypersonic Systems and Technologies*. Norfolk, Virginia.

Fiorentini, L., Serrani, A., Bolender, M.A., and Doman, D.B. (2009). Nonlinear robust adaptive control of flexible air-breathing hypersonic vehicles. *AIAA Journal of Guidance, Control, and Dynamics*, 32(2), 401–416.

Kokotovic, P.V., O'Reilly, J., and Khalil, H.K. (1986). *Singular Perturbation Methods in Control: Analysis and Design*. Academic Press, Inc., Orlando, FL, USA.

Menon, P.K. (2001). Comment on "robust nonlinear control of a hypersonic aircraft". *AIAA Journal of Guidance, Control, and Dynamics*, 24(1), 143–144.

Parker, J.T., Serrani, A., Yurkovich, S., Bolender, M.A., and Doman, D.B. (2007). Control-oriented modeling of an air-breathing hypersonic vehicle. *AIAA Journal of Guidance, Control, and Dynamics*, 30(3), 856–869.

Schmidt, D.K. and Velapoldi, J.R. (1999). Flight dynamics and feedback guidance issues for hypersonic airbreathing vehicles. In *AIAA Guidance, Navigation and Control Conference*, 859–871.

Shkolnikov, I.A. and Shtessel, Y.B. (2001). Aircraft non-minimum phase control in dynamic sliding manifold. *AIAA Journal of Guidance, Control, and Dynamics*, 24(3), 566–572.

Sigthorsson, D., Jankovsky, P., Serrani, A., Yurkovich, S., Bolender, M.A., and Doman, D.B. (2008). Robust linear output feedback control of an airbreathing hypersonic vehicle. *AIAA Journal of Guidance, Control, and Dynamics*, 31(4), 1052–1066.

Wang, Q. and Stengel, R.F. (2000). Robust nonlinear control of a hypersonic aircraft. *AIAA Journal of Guidance Control and Dynamics*, 23(4), 413–423.

Xu, H. and Ioannou, P.A. (2004). Adaptive sliding mode control design for a hypersonic flight vehicle. *AIAA Journal of Guidance Control and Dynamics*, 27(5), 829–838.