Nonlinear Control of a Airbreathing Hypersonic Vehicle

François Poulain* Hélène Piet-Lahanier* Laurent Serre†

Hypersonic flight based on airbreathing propulsion like Scramjet is a challenging research subject in aerospace science, with strong difficulties related to —but not only— control design. The present work establishes a realistic model of such a vehicle, and summarizes difficulties to overcome. Then, a nonlinear control law design is proposed in a suitable choice of coordinates. The control strategy, relying on Lyapunov theory, aims at stabilizing phugoid motion and then backstepping the wished attitude behavior. The results obtained are illustrated by simulation of a realistic model, with a vehicle trajectory varying from Mach 4 at 20 km to Mach 8 at 30 km.

Nomenclature

h	Altitude	m	$\Lambda(V)$	Speed factor	
V	Velocity	m.s^{-1}	$\rho(h)$	Air density	see (1)
γ	Flight path angle	rad	m	Body mass	kg
θ	Attitude	rad	J	Inertial momentum	kg.m^2
α	Angle of attack (AoA)	$\theta - \gamma$	g_t	Earth's gravity	9.8 m.s^{-2}
q	Pitch rate	rad.s^{-1}	$\mathcal{M}$	Mach number	$V/300$
δ	Angle of elevators (AoE)	rad	ε	Air capturing area	m^2
$S(V)$	Similitude factor	see (3)	Q_c	Fuel flow	kg.s^{-2}
C_m	Momentum coefficient		Φ_i	Air to fuel ratio	
C_l	Lift coefficient		f_s	Stoichiometric factor	1/34.572
C_d	Drag coefficient		$I_{\rm sp}$	Specific impulse	s
$l_{\rm ref}$	Reference length	6.9 m	T	Thrust	N
$S_{\rm ref}$	Wing area	7 m	η	Part of controlled thrust	

I. Introduction

It is admitted that many fields relied to space activities (strategics, economics) benefit from increased cruising speed. For sufficiently high speed, the effect of Coriolis acceleration becomes manifest and thus, makes access to space easier. This invites us to reconsider what space vehicles launch might be and, for example, give the final touch to the end of a high speed atmospheric cruising flight.

Hypersonic airbreathing vehicle technology is among others evolving possibilities for space vehicles. This constitutes a major research subject at Onera,[1] currently studied throughout the LEA research program.[2] Numerous improvements on this challenging technique have been achieved by various teams. Nevertheless, noticeable technological difficulties affect their potential application: (i) how to increase cruising speed without dramatically decrease specific impulse and lift-to-drag ratio; (ii) how to design materials able to ensure rigidity of structure under hard constraints; (iii) how to ensure a robust vehicle control. This is precisely the topic we discuss in the present work.

The control of this kind of vehicle has been addressed in several ways, relying on linear control theory (see Ref. 3 and references inside for an overview), dynamic inversion[4] and sliding mode control.[5,6] Linear

*Onera Dprs, 29 avenue de la division Leclerc, BP72 92322 Châtillon Cedex, France.
†Onera Dprs, Chemin de la Hunière, 91761 Palaiseau Cedex, France.

control offers a simple and efficient way to locally stabilize most of (stabilizable) dynamics process, with large possibilities of perfect tuning. However, aerospace systems are often supposed to operate in a wide range of multidimensional state excursions. This suppose to investigate controller interpolation. This may leads to local instability and makes the global behavior study complex. Dynamic-inversion based control laws allow to handle these difficulties. Nevertheless, they lead to complex control structures, embedding huge amount of information in the controller, usually not available in practice. From this point of view, sliding mode control provide a way to control the vehicle addressed here which override these difficulties. However this method is prone to introduce chattering —unhealthy high frequency actuators excitation— which strongly diminishes its efficiency.

An opening to overcome these limitations is offered by a Lyapunov-based nonlinear control, since it potentially provides a certain degree of robustness and some optimality properties.[7] Indeed, robustness of Lyapunov control is obtained if part of speculative *a priori* information is overcome by more reliable structural information. Moreover, Lyapunov theory allows us to use the quasi-full range of the vehicle possibilities, and thus makes it possible to take into account potential saturations. This work aims at providing a model of such hypersonic waverider and then studying stability and nonlinear stabilization, by control Lyapunov function, of a cruising trajectory described in the vertical plane.

The paper is organized as follows. Firstly, we describe our model structure and different general considerations made during our modelling (Section II). The aerodynamic model is detailed and shown to be close to the one in Ref. 8. We next describe how some known specificities of the vehicle are handled by the nonlinear control design. Then, we study the stability of the vehicle, in order to give tools ensuring the stabilization along a trajectory given in terms of speed and altitude (Section III). Next, in Section IV, we detail the way we deal with a more complete behavior knowledge, leading to the designed controller. Finally, simulation results are presented to illustrate the performance and the robustness of the approach (Section V).

II. Modelization of the vehicle

A. General considerations

We are addressing the control of a demonstrator vehicle. As a demonstrator, its modelling is not complete, which is why in the following we describe a general *behavior model*, to be controlled using robust methods w.r.t. model uncertainties.

1. Propulsion vs *aerodynamics interactions*

One specificity of hypersonic waveriders is the strong interaction between the propulsion system and the aerodynamic behavior. This can be detailed throughout few relevant phenomena:

- The vehicle is designed so that its intrados is the propulsion's compression system. Then, the aerodynamic attitude of the vehicle's body —here, the angle of attack (AoA)— plays a prominent role on the air flow circulating throughout the propulsion system.
- The AoA modifies the homogeneity of the flow in the Scramjet, and this impacts the direction which the heat fluxes will take in the nozzle. The exhaust fluxes affect also the aerodynamic behavior of the vehicle's rear.
- Given that the propulsion's direction varies with the state, and nozzle configurations (in addition to other pre-cited phenomena), propulsion is assumed to generate a perturbation momentum which destabilizes the vehicle.

An empirical assumption made is to model propulsion's effect as a simple thrust acting on the velocity axis of the vehicle. We argue this will have no influence on rotational dynamics. First of all, neglecting the influence of propulsion on lift is motivated by the fact that thrust is mainly supposed to compensate drag, *i.e.* lift divided by drag-to-lift ratio; then assuming a perturbed direction would led to a negligible lift action. Next, intrados of vehicle, compressing the hypersonic flow, generates a particularly large pitching momentum which dominates dramatically propulsion's effect.

This momentum induced must also be compensated by huge control surfaces. In a control perspective, such a feature is known to induce a non-minimum phase behavior. The latter entails to restrict possibilities on controller design.[3,4,9] In Section III, a variable change is proposed to handle this problem.

Assumption 1. Propulsion is assumed to have the same direction than the velocity vector of vehicle's center of mass.

Remark 1. Neglecting these propulsion's effect is a simplification motivated by the lack of information to date on these phenomena. It is worth noting that if this had been perfectly known, there would be no obstruction to take it into account in the present work.

2. Hypersonic speed

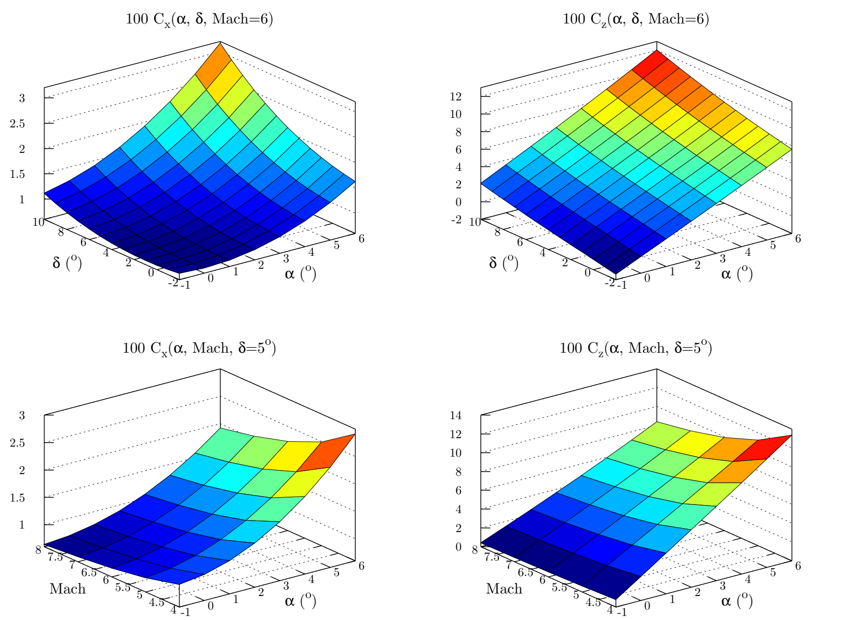

Figure 1. Aerodynamic forces coefficients with respect to AoA α, AoE δ, and Mach number. Particularity of hypersonic wave riders compared to more conventional planes is the dependency of forces with respect to AoE. Effect of high Mach number is a smooth monotonic decreasing of coefficient values.

The idea of a vehicle flying with high speed in the atmosphere led us to consider some specificity in our aerodynamic modelling. Aerodynamic forces are usually modelled as a product between dynamic pressure —which depends of relative air velocity and air density—, and a coefficient term reflecting wetted surface of the wing. This coefficient depends on wing's relative geometry w.r.t. air flow direction, itself depending from AoA, angle of elevators (AoE) and Mach number, as Figure 1 illustrates.

It is generally admitted in the hypersonic domain that these coefficients decrease w.r.t. Mach number. Here we rather use a "scaled" aerodynamic forces model, including a similitude factor $S(V)$. This kind of modelling is in fact motivated by analogy with Newtonian approximation; similar assumption could be to assume a Mach-independent lift-to-drag ratio.

The second principal effect of a hypersonic speed in a control perspective is large uncertainties about aerodynamics coefficients. This motivates us to look for control law which is qualitatively robust to this kind of uncertainties.

3. Limitation of angle of attack

Too high AoA may lead to the deterioration of the propulsion behavior or to the complete destruction of the vehicle. Then, AoA and also pitch velocity should be restricted within bounds to insure a safe flight.

B. Aerodynamics modelling

Atmosphere was modelled using a density function

$$\rho(h) = \exp(0.63 - 1.54 \cdot 10^{-4} h). \tag{1}$$

Aerodynamics was identified from computational results visible on Figures 1 and 2. Following the considerations given about hypersonic aerodynamics, we considered a model such as

$$F_\star = \frac{1}{2}\rho(h) V^2 S_{\mathrm{ref}} S(V) C_\star(\alpha, \delta), \tag{2}$$

where $C_\star$ is an aerodynamic coefficient varying with angle of attack and AoE. The air speed influence $S(V)$ is described in hypersonic domain as

$$S(V) = \frac{1}{0.3 + 0.13\mathcal{M}}, \tag{3}$$

with $V^2 S(V)$ being a monotonic increasing function[a]. We modeled lift coefficient C_l as an affine function of AoA and AoE, and drag such as

$$C_l = C_{l0} + C_{l\alpha}\alpha + C_{l\delta}\delta, \qquad C_d = C_{d0} + C_{dl}C_l(\alpha,\delta)^2. \tag{4}$$

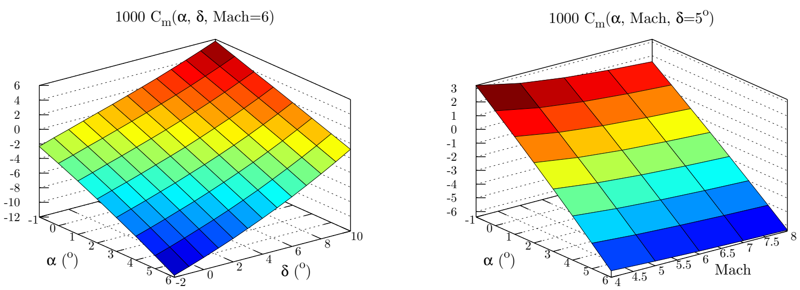

Figure 2. Aerodynamic momentum coefficient with respect to AoA α, AoE δ, and Mach number. Results presented are significantly close to traditional fixed wing vehicle's behavior.

As shown on Figure 2, aerodynamic momentum pitch coefficient is roughly independent of Mach number. Then we modelled the momentum as

$$M_\star = \frac{1}{2}\rho(h)V^2 S_{\text{ref}} l_{\text{ref}} C_\star(\alpha,\delta,q), \text{ with } C_m = C_{m0}(\alpha,q) + C_{m\delta}(\alpha)\delta. \tag{5}$$

C. Propulsion modelling

Scramjet propulsion relies to very complex phenomena to be modelled, and its strong integration to the vehicle's body establishes the need for the propulsion physics to be taken into account at the controller level. Here, we use semi-empiric considerations to model three major phenomena:

- Air inlet. The vehicle's state influences the airflow.
- Combustion. Thrust strongly depends on the quantity of fuel injected, and on the air-to-fuel ratio.
- Efficiency. For a given fuel flow, one can assume that above a certain speed limit, speed increment damages thrust capabilities.

Figure 3 illustrates this kind of modelling w.r.t. fuel flow and AoA for a fixed airspeed, and details our efficiency model w.r.t. speed and air-to-fuel ratio Φ_i. In our work, the control variable considered is the available propulsion ratio.

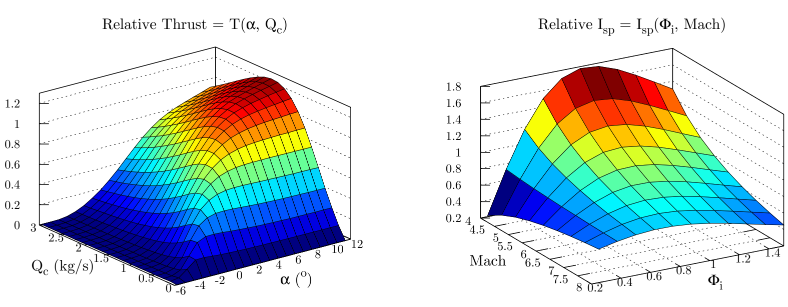

Figure 3. Thrust force, variation w.r.t. AoA α and fuel flow rate Q_c. Specific impulse, variation w.r.t. Mach and air-to-fuel ratio Φ_i. This modelling lead to strong nonlinear behavior, when is considered fuel flow rate or air-to-fuel ratio as controlling input.

Let the thrust be given from specific impulse

$$T = g_t Q_c I_{\text{sp}}(\mathcal{M}, \Phi_i). \tag{6}$$

[a] In the case of a subsonic aircraft, we should simply get $S(V) = 1$. For hypersonic flows, other similitude function may be Prandtl-Glauert rule $S(V) \propto \frac{1}{\sqrt{\mathcal{M}^2 - 1}}$.

As

$$Q_c = f_s Q_a = \Phi_i f_s \rho(h) V \bar{\varepsilon}(\alpha, \mathcal{M}), \tag{7}$$

we can write thrust as

$$T = g_t f_s \rho(h) \bar{\varepsilon}(\alpha, \mathcal{M}) V \Phi_i I_{\mathrm{sp}}(\mathcal{M}, \Phi_i). \tag{8}$$

Assuming that specific impulse can be modelled as a product $I_{\mathrm{sp}}^{V}(\alpha) I_{\mathrm{sp}}^{\Phi_i}(\Phi_i)$, and noticing that the Scramjet is usually designed to keep the term $\bar{\varepsilon}(\alpha_0, \mathcal{M}) V I_{\mathrm{sp}}^{V}(V)$ constant for all α_0, thrust model is thus

$$T = \rho(h) \varepsilon(\alpha) \eta, \qquad \eta \in [0; 1] \tag{9}$$

where $\eta \propto \Phi_i I_{\mathrm{sp}}^{\Phi_i}(\Phi_i)$ is a bounded propulsion characteristic which is assumed to be locally invertible, whereas $\rho(h)\varepsilon(\alpha)$ is the maximal available thrust.

Remark 2. Assuming η as a controlled parameter requires certain knowledge of the air flow rate passing through the propulsion and, so, a suitable description of the characteristic $\Phi_i I_{\mathrm{sp}}^{\Phi_i}(\Phi_i)$. The latter is possible since extensive testing of propulsion is usually realized during vehicle's conception. Using our propulsion modelling, this characteristic is, in fact, a unimodal function, which actually limits the propulsion operating range (from origin to a maximum).

D. Rigid body dynamics

The design of the control is made under the following assumptions.

Assumption 2. Mass variations are negligible during flight.

Assumption 3. Gravity is assumed to be a constant and Coriolis force due to earth's curvature is assumed to be a constant gravity perturbation; *i.e.* $g = g(h_0, V_0)$ for a given reference (h_0, V_0).

We restrict ourselves to the vertical plane. If $\boldsymbol{R}_{0/\mathrm{A}}$ is the orthogonal matrix transforming a velocity related frame to the inertial frame, we can write

$$\dot{\widehat{\boldsymbol{R}_{0/\mathrm{A}} \boldsymbol{V}_G}} = \boldsymbol{R}_{0/\mathrm{A}} \dot{\boldsymbol{V}}_G + \frac{\partial \boldsymbol{R}_{0/\mathrm{A}}}{\partial \gamma_{\mathrm{G}}} \boldsymbol{V}_G \dot{\gamma}_{\mathrm{G}} = \boldsymbol{R}_{0/\mathrm{A}} \frac{\boldsymbol{F}}{m}, \tag{10}$$

and left-multiplying (10) by $\boldsymbol{R}_{0/\mathrm{A}}^{\top}$ leads to the expression of the velocity dynamics (V, γ).

Moreover, since the dynamics modelling is restricted to the plane $(O, \boldsymbol{x}_0, \boldsymbol{z}_0)$, the rotational dynamics is straight forward. Hence the complete modelling of the vehicle is defined in vertical plane by

$$\dot{h} = V \sin\gamma, \tag{11a}$$

$$m\dot{V} = T(\alpha, h, \eta) - \frac{1}{2}\rho(h) V^2 S_{\mathrm{ref}} S(V) C_d(\alpha, \delta) - mg\sin\gamma, \tag{11b}$$

$$mV\dot{\gamma} = \frac{1}{2}\rho(h) V^2 S_{\mathrm{ref}} S(V) C_l(\alpha, \delta) - mg\cos\gamma, \tag{11c}$$

$$\dot{\theta} = q, \tag{11d}$$

$$J\dot{q} = \frac{1}{2}\rho(h) V^2 S_{\mathrm{ref}} l_{\mathrm{ref}} C_m(\alpha, \delta, q). \tag{11e}$$

This model is differs from those used to control planes or missiles. Firstly, the thrust is no more an input variable but a nonlinear function of the state; secondly, as it was argued, the pointed dependency of aerodynamics forces w.r.t. fins' angle control is not neglected here as often done elsewhere.

III. Model study

The model of the vehicle has been established in the preceding Section. We look for Lyapunov control function to globally stabilize the vehicle on a given reference (h_0, V_0) using the controls (η, δ). Our controller is structured in the following way.

First, we have to find some coordinates to make appear a bloc triangular structure. Many classical control designs fail to include the dependency in δ in the γ dynamics. The center of mass path angle is then *not* a suitable output to be controlled on such a vehicle.

Next, we will show that the output block (h, V, γ) of the system is, in fact, a slightly damped stable nonlinear oscillator, for which we can design a gradient control depending on θ and the propulsive balance. This kind of controlling a flight is in fact close to what is usually practiced in manual control, and the type of the proposed control law is known to be efficient, robust, and able to handle saturations.

A. Feedback form of the vehicle

We look for a reduction point, *i.e.* a point D on the body where the elevators do not influence its vertical speed at first order. Using composition law, dynamics at this point are given by

$$\dot{\widehat{\boldsymbol{R}_{0/\mathrm{D}}\boldsymbol{V}_D}} + \dot{\widehat{\boldsymbol{GD} \wedge \Omega_{\mathrm{Body}/0}}} = \boldsymbol{R}_{0/\mathrm{D}}\boldsymbol{R}_{\mathrm{D/A}}\frac{\boldsymbol{F}}{m}. \tag{12}$$

For the sake of simplicity we introduce Assumption 4, easily satisfied for fixed wings flight vehicles.

Assumption 4. The general movement of the vehicle's body is mainly translational. That is $\boldsymbol{R}_{\mathrm{D/A}} \approx \mathbf{I}$.

Moreover, we neglect a frame rotation effect, that is $\boldsymbol{R}_{\mathrm{D}/0}\boldsymbol{GD} \wedge \dot{\boldsymbol{q}} \approx \boldsymbol{GD} \wedge \dot{\boldsymbol{q}}$. Then, if $\boldsymbol{GD} = l_e \boldsymbol{x}_e$, we obtain the (V, γ) dynamics of the point D in inertial frame

$$m\dot{V} = T(\alpha, h, \eta) - \frac{1}{2}\rho(h)V^2 S_{\mathrm{ref}} S(V) C_d(\alpha, \delta) - mg\sin\gamma, \tag{13a}$$

$$mV\dot{\gamma} = \frac{1}{2}\rho(h)V^2 S_{\mathrm{ref}} S(V) C_l(\alpha, \delta) - mg\cos\gamma + ml_e\dot{q}. \tag{13b}$$

Choosing l_e as the lever

$$l_e = -\frac{1}{l_{\mathrm{ref}}}\frac{J}{m}\frac{\partial C_l}{\partial \delta}\frac{\partial \delta}{\partial C_m} S(V), \tag{14}$$

should annihilate δ influence on γ dynamics, since δ appears linearly in C_l and C_m. This variable change is very similar to those obtained using a singular perturbations theory argument.[10]

Remark 3. Two points must be underlined in (14): (i) a hypersonic flight characteristic, the similitude factor, introduces a V-dependency in the change of variable; (ii) the coefficients $\frac{\partial C_m}{\partial \delta}$ and $\frac{\partial C_l}{\partial \delta}$ can be variable according to the modelling level required. Then, this variable change can evolve. Since these complications reduce the interest of this method, we consider in this work a fixed lever distance for a given reference speed. Then we assume that the remaining induced perturbation would be negligible.

Let $e = \frac{1}{m}(T(\alpha, h, \eta) - \frac{1}{2}\rho(h)V^2 S_{\mathrm{ref}} S(V) C_d(\alpha, \delta))$ be the propulsive balance. By reducing γ dynamics, the vehicle's behavior becomes

$$\dot{h} = V\sin\gamma, \tag{15a}$$

$$\dot{V} = e - g\sin\gamma, \tag{15b}$$

$$\dot{\gamma} = \frac{\rho(h)(\Lambda_0(V) + (\theta - \gamma)\Lambda(V)) - g\cos\gamma}{V}, \tag{15c}$$

$$\dot{\theta} = q, \tag{15d}$$

$$J\dot{q} = \frac{1}{2}\rho(h)V^2 S_{\mathrm{ref}} l_{\mathrm{ref}} C_m(\alpha, \delta, q), \tag{15e}$$

where Λ and Λ_0 are monotonic increasing functions. Without loss of generality[b], we consider $\Lambda_0 = 0$ in the following. This last system is a lower block-triangular system: a slightly-damped nonlinear oscillator (h, V, γ) controlled by e and θ, called *phugoid*;[11] the attitude θ and the pitch q controlled by δ.

Let us focus on phugoid motion, with $e = 0$ and $\theta = \theta_0$. The equilibrium of the system is defined by set of couples (h_0, V_0) verifying

$$\rho(h_0)\theta_0\Lambda(V_0) = g(h_0, V_0), \tag{16}$$

then for a θ_0 given, there is not an unique equilibrium point but a manifold[c] linking altitude to airspeed. Given the range of our study (under satellization speed), we assume here the functions ρ, Λ and g get strictly positive values.

[b]This is true in particular when $S(V) = 1$; but all calculus presented in this section can be rewritten considering $\Lambda_0(V) \neq 0$.
[c]In the subsonic case, this manifold verify constant dynamic pressure.

B. Stability of phugoid motion

Stability of Zhukovskii oscillator[12] (simplified (V, γ) dynamics) was already established and exploited for designing nonlinear control.[10,13] This latter work can, in fact, be extended to take into account altitude's dynamics[d] of the phugoid motion.

Proposition 1 (Stability of phugoid motion)**.** *Let a hypersonic flight characterized by a fixed attitude* $\theta = \theta_0 > 0$*, a null propulsive balance* $e = 0$ *and the dynamics*

$$\dot{h} = V \sin\gamma, \qquad \dot{V} = -g \sin\gamma, \qquad \dot{\gamma} = \frac{\rho_0(\theta_0 - \gamma)\Lambda(V) - g\cos\gamma}{V}. \tag{17}$$

Then there exists a couple (h_0, V_0) *verifying* $\rho_0 \Lambda(V_0) = g/\theta_0$ *such that the flight is stable around the equilibrium* $(h_0, V_0, 0)$ *for all initial state* $(h_i, V_i, \gamma_i) \in \mathbb{R} \times \times \mathbb{R}^+ [-\pi; \pi]$.

Proof. Consider Lyapunov function

$$W(h, V, \gamma) = \rho_0 \theta_0 \int_{V_0}^{V} \Lambda(v) - \Lambda(V_0) dv + gV(1 - \cos\gamma), \tag{18}$$

which is positive semi-definite, since Λ is a monotonic increasing function. Its derivative w.r.t. time along trajectories verifies

$$\dot{\widehat{W(h, V, \gamma)}} = -g\rho_0 \Lambda(V) \gamma \sin\gamma \leq 0, \tag{19}$$

Since $g\rho_0\Lambda(V)$ is positive for all $V \in \mathbb{R}^+$, this proves stability. From LaSalle theory,[14] we thus know that there is a couple (h_0, V_0) such that asymptotic convergence of $(h, V) \rightarrow (h_0, V_0)$ is verified. □

C. Sufficient conditions for stabilization

The result given by Proposition 1 is a *strong* qualitative property to be exploited in control design. Stability of phugoid motion implies that dynamic drift does not have necessarily to be compensated in the control law. Efficient stabilization requires only

$$\operatorname{sign} e = -\operatorname{sign}(\rho_0\theta_0\Lambda(V) - g\cos\gamma), \qquad \operatorname{sign}(\theta - \theta_0) = -\operatorname{sign}(gV\sin\gamma). \tag{20}$$

This makes it possible to ensure global stability of flight using a bounded range of propulsive balance or attitude. The latter is important in order to maintain the integrity of the vehicle and the viability of its control law. Moreover, the control law does not need to embed a lot of *a priori* information —this is particularly true in the case of θ which should depend only from $\sin\gamma$— and this provides certain qualitative robustness of the control law.

D. Influence of a heterogeneous atmosphere

The Proposition 1 establishes a stability result in the context of a homogeneous atmosphere. We should study impact of a heterogeneous atmosphere on stability of phugoid motion. Let $\rho(h)$ replace ρ_0 in our autonomous behavior model (17), ones obtain

$$\dot{\widehat{W(h, V, \gamma)}} \leq g\theta_0 \Lambda(V)(\rho(h) - \rho_0)\sin\gamma. \tag{21}$$

This drift can be partially eliminated. Consider the Lyapunov function

$$W_h(h, V, \gamma) = W(h, V, \gamma) + \theta_0 g \frac{\Lambda(V_0)}{V_0} \int_{h_0}^{h} \rho(h_0) - \rho(s) ds. \tag{22}$$

This function is positive semi-definite, since the density gradient of atmosphere remains strictly negative. Its derivative along trajectories is

$$\dot{\widehat{W_h(h, V, \gamma)}} = -g\rho(h)\Lambda(V)\gamma\sin\gamma + (\rho(h) - \rho(h_0)) \left(\Lambda(V) - \Lambda(V_0)\frac{V}{V_0} \right) g\theta_0 \sin\gamma. \tag{23}$$

[d] The influence of a heterogeneous atmosphere on stability is developed later in the Section III.

The second remaining term of third order should be locally dominated $-g\rho(h)\Lambda(V)\gamma\sin\gamma$. One can notice that if $\Lambda(V) \propto V$, then the stability result should be recovered. In fact, effect of hypersonic speed modelled by $S(V)$ leads to this kind of behavior: assuming l_e sufficiently small, a similitude factor such as Prantl-Glauert rule leads to $\lim_{l_e \to 0} \Lambda(V) = S(V)V^2$ and is roughly proportional to V for $\mathcal{M} > 3$. For these reasons, the effect of atmosphere's heterogeneity can be summarized as:

- influence of atmosphere's heterogeneity on stability is light, especially compared to control effort;
- using additional assumptions, stability result can be recovered with heterogeneous atmosphere;
- taking into account atmosphere's heterogeneity in Lyapunov function will not change main control strategy: since $\dot{h}$ is not directly actuated, we need supplementary factors in Lyapunov function in order to be able to dominate the perturbation and stabilize h to a given reference.

E. Airbreathing propulsion

As described in Section II, our aerobic propulsion is a state-dependent function, which allows various way of choose what should the best suitable input. In the context we developed here, we will argue that η seems to be an interesting control input for different reasons:

- The fact that η appears linearly in the model leads to quite simple control law, verifying qualitative robustness. This affine behavior is no longer met if we consider fuel flow rate Q_c or air-to-fuel ratio Φ_i.
- In practice, the operating range of η is roughly linear w.r.t. Φ_i which is an significant physical parameter to qualify combustion.
- Locally, under some standard assumption, for a given fixed control η_0 the vehicle remains stable.

This last stability result is similar to a case of anaerobic propulsion. Our choice of η as input variable leads to the propulsive balance equation

$$e(h, V, \gamma, \theta) = \frac{1}{m}\rho(h)\varepsilon(\alpha)\left(\eta - \frac{C_d(\alpha, \delta)}{2\varepsilon(\alpha)} S_{\text{ref}} S(V) V^2\right), \tag{24}$$

with η as a control parameter. From this, deriving $W(h, V, \gamma)$ along trajectories gives

$$\dot{\widehat{W_h(h, V, \gamma)}} = \dot{\widehat{W_h(h, V, \gamma)}}\Big|_{e=0} + (\rho_0\theta_0\Lambda(V) - g\cos\gamma)\,\frac{\rho(h)\varepsilon(\alpha)}{m}\left(\eta - \frac{C_d(\alpha, \delta)}{2\varepsilon(\alpha)} S_{\text{ref}} S(V) V^2\right). \tag{25}$$

For all admissible δ_0, the characteristic $C_d(\alpha, \delta)/\varepsilon(\alpha)$ is an unimodal function of α with a minima in the normal operating range. A standard argument from singular perturbation theory[10,15] allows us to handle $C_d(\alpha, \delta)/\varepsilon(\alpha)$ as a quasi-constant parameter in context of phugoid motion. Then, assuming $\cos\gamma \approx 1$, the vehicle will be stabilized at the value V_0 which satisfies $\eta_0 = \frac{1}{2} C_d(\alpha, \delta)/\varepsilon(\alpha) S_{\text{ref}} S(V_0) V_0^2$. This result is provided by the fact that $\Lambda(V)$ and $S(V)V^2$ are both monotonically increasing functions. Stability is then a logical consequence of energy dissipation by drag.

IV. Realization of the control law

Preceding section was dedicated to study phugoid motion's properties and how to take into account these properties in order to define a controller. To establish the final control law, we also need (i) to ensure convergence of h to its reference; and (ii) to back-step the result obtained in order to ensure the stabilization of the model (15).

A. Stabilization of $h \to h_0$

Stabilization of h is made possible by using new term factor in the Lyapunov function. From the model structure, we suggest to use forwarding,[16] and a modified version of forwarding described in Ref. 17.

1. *Forwarding*

Consider $\psi_e(gh+\frac{1}{2}V^2-E_0)$, with $E_0 = gh_0+\frac{1}{2}V_0^2$ defined as an total energy objective term, and $\psi_e : \mathbb{R} \to \mathbb{R}^+$ a Lipschitz positive definite function. Its derivative along trajectories verifies

$$\psi_e \overset{\cdot}{\overbrace{\left(gh + \frac{1}{2}V^2 - E_0\right)}} = \psi_V' \left(gh + \frac{1}{2}V^2 - E_0\right)\left(gV\sin\gamma + V\left(e - g\sin\gamma\right)\right), \tag{26}$$

$$= \psi_V' \left(gh + \frac{1}{2}V^2 - E_0\right) V e\left(h, V, \gamma, \theta\right), \tag{27}$$

which is affine in control. From this, considering the propulsive balance controlled, stability of dynamic drift is unchanged (energy conservation of non-dragged motion). This allows us to stabilize altitude to a given reference, by modifying energy level *via* the thrust.

2. *Forwarding* $\mathrm{mod}\{L_gV\}$

The use of forwarding is a physically nice way to handle altitude stabilization but, in this case, is conservative. This is an interesting feature if we aim at saving energy, but if our interest lies in reaching a specific value of altitude, this will not prove very efficient. For that purpose, notice the dynamic of h, which is $\frac{1}{g}\frac{\partial W_h}{\partial \gamma}$: a factor allowing us to eliminate any h-based control objective *via* θ.

Consider the term $\psi_\gamma(h-h_0)$, with $\psi_\gamma : \mathbb{R} \to \mathbb{R}^+$ Lipschitz and positive definite. Its derivative along trajectories is

$$\overset{\cdot}{\overbrace{\psi_\gamma(h-h_0)}} = \psi_\gamma'(h-h_0)V\sin\gamma = \psi_\gamma'(h-h_0)\frac{1}{g}\frac{\partial W_h}{\partial\gamma}, \tag{28}$$

Since the term $\frac{\partial W_h}{\partial \gamma}$ is in factor, the cross term will be easily eliminated by defining a suitable control law for θ. Convergence of h to a given reference is thus provided by a LaSalle argument.[14]

B. Guidance control choice

It appears that there is a wide variety of implementation of controllers stabilizing phugoid motion evolving in $(h, V, \gamma) \in \mathbb{R}\times\mathbb{R}^+\times[-\pi;\pi]$. Here we summarize the structure of our implemented controller. We consider the Lyapunov function

$$W_0(h, V, \gamma) = W_h(h, V, \gamma) + \psi_\gamma(h-h_0) + \psi_e(gh + \frac{1}{2}V^2 - E_0), \tag{29}$$

with $E_0 = gh_0 + \frac{1}{2}V_0^2$. The functions $\psi_\star$ are defined as

$$\psi_\gamma(s) = \int_0^s \mathrm{sat}(k_{\psi_\gamma} r, \underline{h_\theta}, \bar{h_\theta})dr, \qquad \psi_e(s) = \int_0^s \mathrm{sat}(k_{\psi_e} r, \underline{E}, \bar{E})dr, \tag{30}$$

where $\mathrm{sat}(r, \underline{r}, \bar{r}) = \min(\max(r, \underline{r}), \bar{r})$. Its derivative along trajectories is

$$\begin{aligned}\overset{\cdot}{\overbrace{W_0(h, V, \gamma)}} = \overset{\cdot}{\overbrace{W_0(h, V, \gamma)}}\Big|_{e=0,\,\theta=\theta_0} &+ V\sin\gamma\left(g\rho(h)(\theta-\theta_0)\frac{\Lambda(V)}{V} + \psi_\gamma'\right)\\ &+ \left(\rho_0\theta_0\Lambda(V) - g\cos\gamma + \psi_e' V\right)\frac{\rho(h)\varepsilon(\alpha)}{m}\left(\eta - \frac{C_d(\alpha,\delta)}{2\varepsilon(\alpha)}S_{\mathrm{ref}}S(V)V^2\right).\end{aligned} \tag{31}$$

In order to make the derivative negative, we choose controls as

$$\theta_c = \theta_0 - \frac{V}{g\rho(h)\Lambda(V)}\psi_\gamma'(h-h_0) - k_\gamma\gamma, \tag{32a}$$

$$\eta_c = \frac{C_d(\alpha,\delta)}{2\varepsilon(\alpha)}S_{\mathrm{ref}}S(V)V^2 - k_e\frac{1}{m}\rho(h)\varepsilon(\alpha)\left(\rho_0\theta_0\Lambda(V) - g\cos\gamma + V\psi_e'(gh + \frac{1}{2}V^2 - E_0)\right), \tag{32b}$$

$$\eta = \mathrm{sat}(\eta_c, \underline{\eta}, \bar{\eta}), \qquad \theta = \mathrm{sat}(\theta_c, \underline{\theta}, \bar{\theta}). \tag{32c}$$

Under arbitrary bounded constraints, this control law will then achieve asymptotic stability of a couple (h_0, V_0). The given bounds must be such that they provide the way to counteract the drifts due to drag and altitude objective ψ'_γ. In fact, this is nothing else than a controllability assumption.

At this stage, the main critical information embedded in the controller is the term $C_d(\alpha, \delta)/\varepsilon(\alpha)$, which can be partially unknown and difficult to measure or reconstruct. Most of the remaining terms are relied to physical laws, directly measurable parameters, or multiplicative functions with known sign. This property is important in order to keep qualitative robustness of the control law.

C. Attitude control

Stabilization of the phugoid motion assumes that θ is a directly available control. Actually, θ can be seen as the output of a slightly damped fast oscillator system. From the triangular structure of the system, we propose to stabilize it by rendering attractive and stable a manifold $\theta_c(h, V, \gamma)$ given by control law (32c). Backstepping[18] provides an efficient way to extend our control law by this manner, and ensuring assignability of the Lyapunov function, given the integrators chain *elevators→angular speed→attitude*.[10,19]

Consider the rotational dynamic

$$\dot{\theta} = q, \qquad \dot{q} = \frac{1}{2J}\rho(h)V^2 S_{\text{ref}} l_{\text{ref}} C_m(\alpha, \delta, q), \tag{33}$$

and the Lyapunov function (29). Its derivative is

$$\dot{\widehat{W_0(h, V, \gamma)}} = \dot{\widehat{W_0(h, V, \gamma)}}\Big|_{\theta=\theta_c} + g \sin\gamma \rho(h)\Lambda(V)(\theta - \theta_c). \tag{34}$$

Then, extending the Lyapunov function as

$$W_1(h, V, \gamma, \theta) = W_0(h, V, \gamma) + \frac{1}{2}(\theta - \theta_c)^2, \tag{35}$$

its derivative is

$$\dot{\widehat{W_1(h, V, \gamma, \theta)}} = \dot{\widehat{W_0(h, V, \gamma)}}\Big|_{\theta=\theta_c} + (\theta - \theta_c)\left(g \sin\gamma \rho(h)\Lambda(V) + q - \dot{\theta}_c\right). \tag{36}$$

If we are able to ensure the convergence of q toward

$$q_c = -\operatorname{sat}(k_q(\theta - \theta_c), \underline{q}, \bar{q}) + \dot{\theta}_c - g \sin\gamma \rho(h)\Lambda(V), \tag{37}$$

the subsystem (h, V, γ, θ) would thus be stabilized, and this achieve the first step of backstepping. The next step is the stabilization of the overall system: extending Lyapunov function as

$$W_2(h, V, \gamma, \theta, q) = W_1(h, V, \gamma, \theta) + \frac{1}{2}(q - q_c)^2, \tag{38}$$

and derivative give

$$\dot{\widehat{W_2(h, V, \gamma, \theta, q)}} = \dot{\widehat{W_1(h, V, \gamma, \theta)}}\Big|_{q=q_c} + (q - q_c)\left(\theta - \theta_c + \frac{1}{2J}\rho(h)V^2 S_{\text{ref}} l_{\text{ref}} C_m(\alpha, \delta, q) - \dot{q}_c\right). \tag{39}$$

Then, finally, a control law stabilizing our vehicle described by (15) is

$$\delta = -\operatorname{sat}(Q, \underline{\delta}, \bar{\delta}) - \frac{C_{m0}(\alpha, q)}{C_{m\delta}(\alpha)} + (\dot{q}_c + \theta_c - \theta)\frac{2J}{\rho(h)V^2 S_{\text{ref}} l_{\text{ref}} C_{m\delta}(\alpha)}, \tag{40}$$

with $Q = \frac{1}{2J}\rho(h)V^2 S_{\text{ref}} l_{\text{ref}} C_{m\delta}(\alpha)(q - q_c)$.

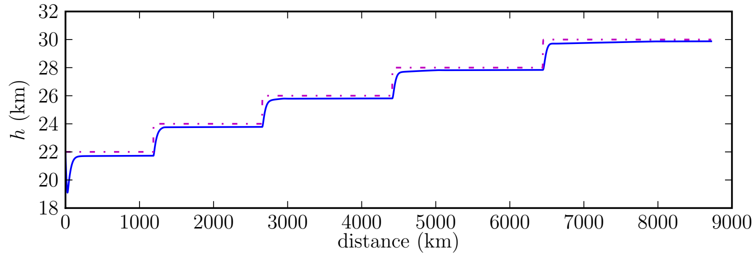

Figure 4. Trajectory simulated during flight.

V. Simulations and comments

A simulation has been made, using the controller (40). The maximal thrust-to-mass ratio of the simulated vehicle is about 3 at Mach 8, with a lift-to-drag ratio about to 3.7. The vehicle's initial mass is 5 000 kg which consist for half of propellant. Measurement of state and actuators (η, δ) are considered perfect. The simulation implements a complex knowledge modelling: aerodynamics comes from table lookup; propulsion is considered asymmetric; a strongly nonlinear air inlet was considered.

In practice, modelling is known to provoke large quantitative errors, especially about forces and momentum prediction. As an example, at Mach 8, the following values were monitored: (i) aerodynamics torque was predicted with a factor 0.8 to 1.5 compared to simulation model, depending on configuration; (ii) drag and lift coefficient were predicted respectively 76% and 68% besides their values used for simulation; (iii) propulsion was predicted 50% besides its value used for simulation. In the control implementation, η is computed as a proportional to Φ_i.

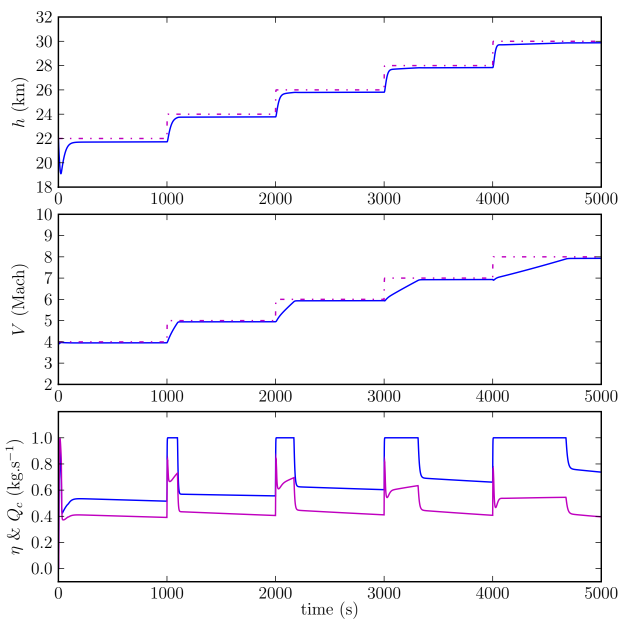

Figure 5. Simulation: convergence of h and V to references.

Figure 4 presents the trajectory of the simulated flight using the control law (40). This trajectory aims at following a succession of references (h_0, V_0). The same controller was used during all the flight. Convergence of altitude and speed with respect to time is presented on Figure 5. As it is shown, large uncertainties on the model leads here to asymptotic errors on the reference followed, but without affecting stability properties. The maximal available thrust is used whenever necessary.

On Figure 6, is shown the behavior of the controlled vehicle subject to an altitude and Mach reference step encountered at $t = 2000$ s. A maximal bound of 8 deg on angle of attack is respected during ascension

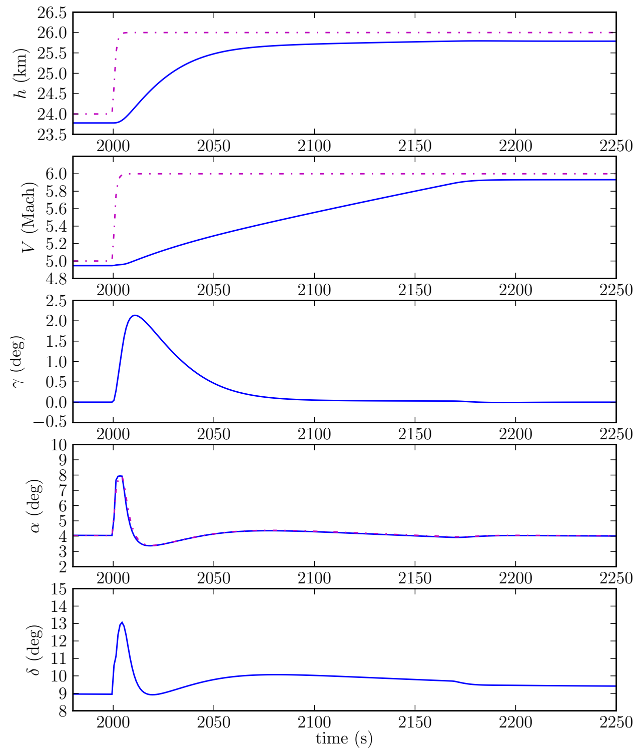

Figure 6. Simulation: zoom on an altitude reference's step.

which actually limits $\dot{\gamma}$. These bounds are chosen to limit thermal fluxes, load factor, or to protect air inlet from an exhibition to hypersonic flow.

The figure 7 illustrates more precisely this α-bounded behavior. The vehicle was actually launched at $h = 22$ km, Mach 4. The initial values $\gamma = -10$ deg and $q = -10$ deg.s^{-1} have been chosen as rather tough initial condition in order to illustrate some extreme behavior. Therefore, the control first increases and stabilizes the attitude in order to use the maximal allowed given excursion value for AoA. This pulling up corrects quickly the flight path angle, until it becomes sufficiently secure to mainly correct altitude. The end of the stabilization is finally achieved in the linear range of the intermediate control function $\mathrm{sat}(\theta_c, \underline{\theta}, \bar{\theta})$ given in (32c). This kind of strong nonlinear bound provides interesting tools to extend attraction domain of the controller, with respect to some viability constraint.

VI. Conclusion

This paper has presented the controller synthesis of the longitudinal mode of a hypersonic vehicle with airbreathing propulsion on a cruising trajectory. To this end, a complete modelling was established, followed by a summary of difficulties to overcome. We argued these difficulties may be viewed as three relevant phenomena:

- The complexity of the propulsion system. We reduced the complexity of the propulsion system by considering as a control the ratio to the maximal available thrust. This variable control appears to be roughly proportional to a characteristic of the propulsion system.

- The non minimum phase behavior. Under the main hypothesis of an affine dependency of lift and torque w.r.t. angle of elevators, we proposed a change of variable suitable to avoid this structural difficulty.

- The quantitative uncertainties on knowledge model. As we argued, the way we should robustify our control structure from quantitative uncertainty consists of making control laws linked to reliable

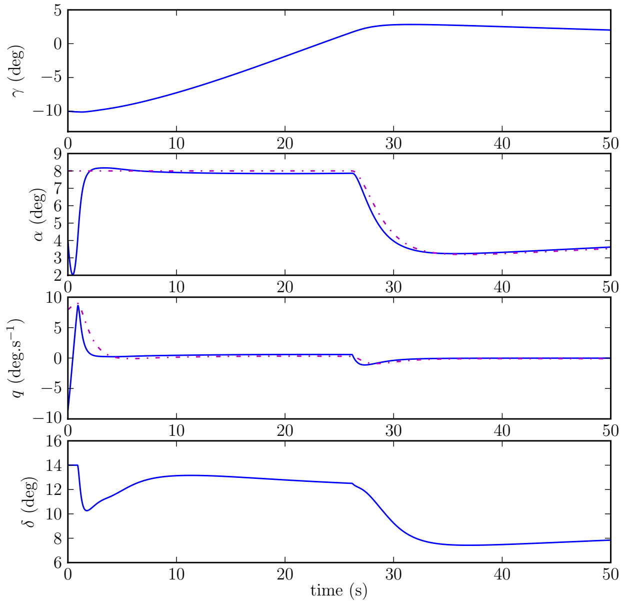

Figure 7. Simulation: zoom on unhealthy launch situation.

information. To this end, our intermediate control law (32c) mainly depends on γ, and h, which are commonly well measured. An additional feature provided by this control law is simplicity: on-line computation is then easily ensured. The control law is also provided with its associated Lyapunov function which gives an appropriate tool to guarantee gain margin w.r.t. model uncertainties, structured or not.

The result obtained is a nonlinear control law which ensure global stability property on all the domain of validity of the model. Complexity of propulsion and aerodynamic structure has been taken into account, and this work might be specialized to vehicles with anaerobic propulsion system, or operating in subsonic domain. This controller offers lot of degrees of freedom; as an example, the controller might be locally identified as the solution of a given optimal control, for example a LQR design.

This comes from a deep analysis of phugoid motion from Lyapunov point of view. This was done using few assumptions which have been discussed. The point of view adopted is to consider flight as a conservative motion, and thus realize a gradient control which allows us to take into account viability properties. To the knowledge of the authors, this property was never exploited to design nonlinear control of waveriders. Gradient control offers the property to be robust with a rational use of control effort: no high gain properties are thus exploited.

The potential limits of this control law are the following: (i) backstepping is an efficient way to extend Lyapunov function and globally control the vehicle. However it reintroduces complexity and does not take advantage of a possible stability of rotational motion; (ii) in the same way, backstepping does not offer efficient ways to deal with actuator saturation, in the end of a control chain; (iii) in spite of qualitative robustness properties, our controller does not ensure a strict asymptotic convergence to the reference: this is a lack of quantitative robustness. These different issues are currently under investigation, as well as the extension of the approach to a 6-DOF flight.

Acknowledgments

The authors are very grateful to Dr. Laurent Praly for his useful wise advices about nonlinear control techniques.

References

[1] Lentsch, A., Bec, R., Deneu, F., Novelli, P., Rigault, M., and Conrardy, J.-M., "Air-Breathing Launch Vehicle Activities in France," *12th AIAA International Space Planes and Hypersonic Systems and Technologies*, Norfolk, Virginia, 2003.

[2] Falempin, F. and Serre, L., "French R&T effort for high-speed airbreathing propulsion Status in 2008," *59th International Astronautical Congress (IAC-08-C4.5.5)*, Glasgow, Scotland, October 2008.

[3] Sigthorsson, D., Jankovsky, P., Serrani, A., Yurkovich, S., Bolender, M. A., and Doman, D. B., "Robust Linear Output Feedback Control of an Airbreathing Hypersonic Vehicle," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 4, 2008, pp. 1052–1066.

[4] Wang, Q. and Stengel, R. F., "Robust Nonlinear Control of a Hypersonic Aircraft," *AIAA Journal of Guidance Control and Dynamics*, Vol. 23, No. 4, 2000, pp. 413–423.

[5] Xu, H. and Ioannou, P. A., "Adaptive Sliding Mode Control Design for a Hypersonic Flight Vehicle," *AIAA Journal of Guidance Control and Dynamics*, Vol. 27, No. 5, 2004, pp. 829–838.

[6] Fidan, B., Mirmirani, M., and Ioannou, P. A., "Flight Dynamics and Control of Air-Breathing Hypersonic Vehicles: Review and New Directions," *12th AIAA International Space Planes and Hypersonic Systems and Technologies*, Norfolk, Virginia, 2003.

[7] Kokotovic, P. and Arcak, M., "Constructive nonlinear control: a historical perspective," *Automatica*, Vol. 37, No. 5, 2001, pp. 637 – 662.

[8] Parker, J. T., Serrani, A., Yurkovich, S., Bolender, M. A., and Doman, D. B., "Control-Oriented Modeling of an Air-Breathing Hypersonic Vehicle," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 3, 2007, pp. 856–869.

[9] Menon, P. K., "Comment on "Robust Nonlinear Control of a Hypersonic Aircraft"," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 1, 2001, pp. 143–144.

[10] Poulain, F., Piet-Lahanier, H., and Serre, L., "Nonlinear Control of a Airbreathing Vehicle: a Backstepping Approach," *IFAC Workshop, Aerospace Guidance, Navigation, and Flight Control Systems*, Samara, Russia, June 2009.

[11] Lanchester, F. W., *Aerodonetics: constituting the second volume of a complete work on aerial flight*, Archibald Constable & co. ltd., London, 1908.

[12] Andronov, A. A., Vitt, A. A., and Khaikin, S. E., *Theory of Oscillators*, Dover, 1987.

[13] Praly, L. and Coron, J. M., "Guidage en rentrée atmospherique," Tech. rep., CNES, 2001.

[14] Khalil, H. K., *Nonlinear Systems*, Prentice Hall, third edition ed., 2002.

[15] Kokotovic, P. V., O'Reilly, J., and Khalil, H. K., *Singular Perturbation Methods in Control: Analysis and Design*, Academic Press, Inc., Orlando, FL, USA, 1986.

[16] Mazenc, F. and Praly, L., "Adding integrations, saturated controls, and stabilization for feedforward systems," *Automatic Control, IEEE Transactions on*, Vol. 41, No. 11, Nov 1996, pp. 1559–1578.

[17] Praly, L., Ortega, R., and Kaliora, G., "Stabilization of nonlinear systems via forwarding mod LgV," *Automatic Control, IEEE Transactions on Automatic Control*, Vol. 46, No. 9, 2001, pp. 1461–1466.

[18] Freeman, R. A. and Kokotović, P. V., *Robust Nonlinear Control Design*, ISBN: 978-0-8176-4758-2, 1996.

[19] Harkegard, O. and Torkel Glad, S., "A backstepping design for flight path angle control," *Decision and Control, 2000. Proceedings of the 39th IEEE Conference on*, Vol. 4, 2000, pp. 3570–3575.