Nonlinear Control of a Airbreathing Hypersonic Vehicle: A Backstepping Approach

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Abstract: Nowadays, airbreathing vehicle's control represents a very motivating challenge, as it includes the respect of some physical constraints, hard nonlinearities in thrust operation, and also a non minimum phase behavior due to fins' size. In this paper we describe methods for simultaneous guidance and control nonlinear laws design relying on Lyapunov functions theory. Some simulation are presented, based on a realistic modelling of such a vehicle.

Keywords: Nonlinear control, Flight control, Lyapunov function, Hypersonic waveriders.

1. NOMENCLATURE

ρ	Air density	$\exp(0.63-1.54\ 10^{-4}h)$
m	Body mass	kg
g_0	Gravity	9.8 m.s^{-2}
J	Inertial momentum	kg.m^2
$\mathcal{M}$	Mach number	$V/300$
l_{ref}	Reference length	6.9 m
f_s	Stoichiometric factor	1/34.572
S_{ref}	Wing area	7 m
h	Altitude	km
V	Velocity	m.s^{-1}
γ	Flight path angle	rad
α	Angle of attack (AoA)	rad
q	Pitch rate	rad.s^{-1}
δ	Angle of elevators	rad
d	Drag factor	$\frac{1}{2m}\rho S_{\text{ref}}\bar{C}_D$
$\Lambda(V)$	Pressure factor w.r.t. V	$\frac{V^2}{0{,}3+0.13\mathcal{M}}$
l	Lift factor	$\frac{1}{2m}\rho S_{\text{ref}}\bar{C}_L$
C_M	Momentum coefficient	
$\bar{C}_L$	Reduced lift coefficient	
$\bar{C}_D$	Reduced drag coefficient	
f	Lift-to-drag ratio	$\frac{l}{d}$
ε	Air capturing area	m^2
Q_c	Fuel flow	kg.s^{-2}
Φ_i	Air to fuel ratio	
I_{sp}	Specific impulse	s
I_{sp}^V	Specific impulse w.r.t. V	$\frac{I_{\text{sp}_0}}{\mathcal{M}-1.371}$
T	Thrust	N
t	Thrust factor	$\frac{T}{m}$
η	Thrust ratio	

2. INTRODUCTION

The various interests of airbreathing propulsion had been identified for a long time in various applications, see *e.g.* Lentsch et al. (2003) which resume airbreathing vehicle's development in Onera, or Falempin and Serre (2008) for a description of LEA project status. As an example, in the missile domain, hypersonic cruise at high altitude tends to insure long dynamic range and decreases interception's risks. In the case of spacecraft applications, the ability to use air-picked oxydizer on a major part of the trajectory could strongly increase the mass ratio to orbital speed (currently oxydizer is close to 75% of the mass of a launcher like Ariane V).

Controlling such a vehicle has been addressed in several ways, relying on linear control theory (see *e.g.* Sigthorsson et al. (2008) and references inside), dynamic inversion, and sliding-modes control (see Wang and Stengel (2000) and Xu and Ioannou (2004)). However, most of these works did not considered hard nonlinearities induced by propulsion system and non minimum phase behavior.

Our work aims at providing a model of such a vehicle, and then studying stability and nonlinear stabilisation by control Lyapunov function along a trajectory described in the vertical plane. Lyapunov theory allows us using the full range of our different actuators, taking in account potential saturations. It is also known to insure certain degree of robustness and optimality properties.

The paper is organized as follow. We firstly describe in Sections 3 and 4 the model structure and the different assumptions made on the modelling structure of the aerodynamics and thrust. Our aerodynamic structure is designed from semi-empiric methods, and the resulting model structure similar to what has been published in Parker et al. (2007). We will next assume in Section 5 that a reference trajectory exists in terms of flight speed and path angle. We aim at designing in Section 6 a closed-loop guidance and control law that will stabilize the vehicle along this trajectory. In this paper, focuses will be put on specificities brought by these waverider concepts, compared with more traditional aircrafts. Finally, examples will be presented in Section 7 to illustrate the performances of the approach.

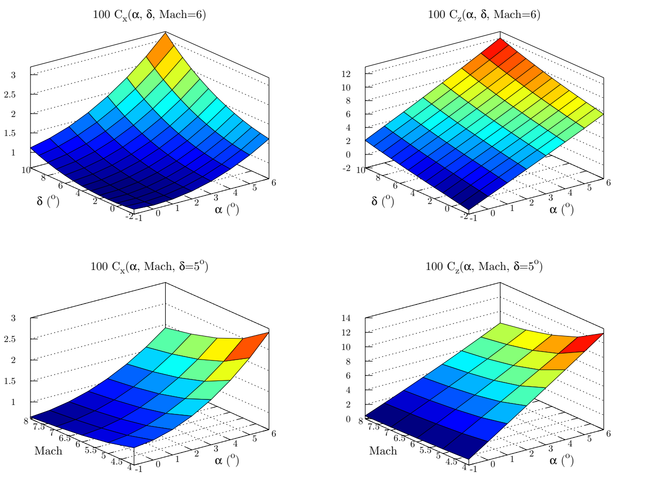

Fig. 1. Aerodynamic forces coefficients with respect to angle of attack, angle of elevators, and Mach number.

3. MODELISATION OF THE VEHICLE'S DYNAMIC

3.1 Aerodynamic forces

Aerodynamic forces are usually modelled as a product of dynamic pressure —which depends of relative air velocity and air density—, and a coefficient term reflecting wetted surface of the wing. This coefficient depends on wing's geometry and relative air flow direction:

$$F_\star = \frac{1}{2}\rho V^2 S_{\text{ref}} C_\star, \tag{1}$$

where $C_\star$ is an *aerodynamic coefficient*. It varies with angle of attack, angle of elevators and Mach number; as Figure 1 illustrates, it is well known in the hypersonic domain that the $C_\star$ coefficients decrease w.r.t. Mach number. Here we rather use an aerodynamic force model like

$$F_\star = \frac{1}{2}\rho(h)\Lambda(V)C_\star(\alpha,\delta), \tag{2}$$

where $\Lambda(V)$ is a monotonic increasing function relying on flow pressure [1]. We will use

$$\Lambda(V) = \frac{V^2}{s_0 + s_1 V}. \tag{3}$$

This kind of modelling is, in fact, motivated by analogy with hypersonic Newtonian approximation, and it will be assumed consequently a Mach-independent lift-to-drag ratio. This property will be exploited by the control presented in the following.

We finally modelled our forces coefficients as a linearly parameterized function in (α, δ) using data obtained from semi-empiric methods (see Figure 1 and 2):

$$C_L = \mathbf{L}_L \begin{pmatrix}1 & \alpha & \delta\end{pmatrix}^\top, \tag{4}$$

$$C_D = \mathbf{L}_D \begin{pmatrix}1 & \alpha & \delta\end{pmatrix}^\top + \begin{pmatrix}\alpha & \delta\end{pmatrix} \mathbf{Q}_D \begin{pmatrix}\alpha & \delta\end{pmatrix}^\top. \tag{5}$$

3.2 Aerodynamic momentum

Similarly to the aerodynamics forces, aerodynamic momentum is usually modelled as

$$M_\star = \frac{1}{2}\rho V^2 S_{\text{ref}} l_{\text{ref}} C_\star. \tag{6}$$

[1] In the case of a subsonic aircraft, we can simply get $\Lambda(V) = V^2$.

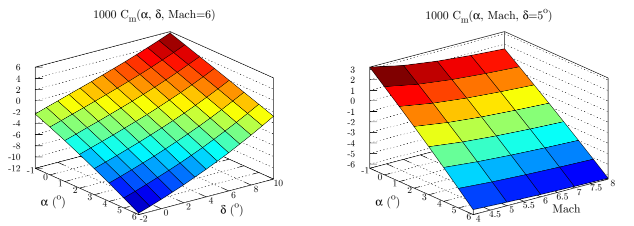

Fig. 2. Aerodynamic moment coefficient with respect to angle of attack, angle of elevators, and Mach number.

As it can be seen on Figure 2, aerodynamic pitch coefficient is roughly independent from Mach, and varies quasi-linearly with angle of attack and angle of elevators. There exists also a damping effect to be added, assumed to be linear in $\frac{l_{\text{ref}}}{V}q$. So we modelled the momentum coefficient as the linearly parameterized function:

$$C_M = C_{M0} + C_{M\alpha}\alpha + C_{Mq}\frac{l_{\text{ref}}}{V}q + (C_{M\delta_0} + C_{M\delta_\alpha}\alpha)\,\delta. \tag{7}$$

3.3 Scramjet thrust

Scramjet propulsion is a very complex phenomenon which will be modelled using a specific impulse function depending on Mach number and air-fuel ratio:

$$T = g_0 Q_c I_{\text{sp}}(\mathcal{M}, \Phi_i). \tag{8}$$

Figure 3 illustrates the input/output behavior of such a model. It appears that specific impulse can itself be split as a product of two terms:

- the dependency on Mach —relying on efficiency— monotonic and decreasing in all hypersonic domain, which can be assimilated to a dependency in V;
- the dependency on air-fuel ratio, an unimodal function passing through a maximum near $\Phi_i = 0.7$.

As

$$Q_c = \Phi_i f_s Q_A = \Phi_i f_s \rho V \bar{\varepsilon}(\alpha, \mathcal{M}), \tag{9}$$

we can rewrite equation (8) as

$$T = g_0 f_s \rho \bar{\varepsilon}(\alpha, \mathcal{M}) \Phi_i I_{\text{sp}}^{\Phi_i}(\Phi_i) V I_{\text{sp}}^{V}(V), \tag{10}$$

but since the Scramjet is usually designed to keep the term $\rho\bar{\varepsilon}(\alpha_0, \mathcal{M})V$ constant, we thus can model thrust as [2]

$$T = m\varepsilon(\alpha) I_{\text{sp}}^V(V)\eta, \qquad \eta \in [0;1] \tag{11}$$

where $\eta \propto \Phi_i I_{\text{sp}}^{\Phi_i}(\Phi_i)$ is a bounded propulsion characteristic which is assumed to be locally invertible. Finally we modelled air-captation model as a locally affine function

$$\varepsilon(\alpha) = \max(\varepsilon_0, \min(\varepsilon_{00} + \varepsilon_{0\alpha}\alpha, \varepsilon_{10} + \varepsilon_{1\alpha}\alpha)), \tag{12}$$

and thrust efficiency w.r.t. Mach as

$$I_{\text{sp}}^V(V) = (i_0 + i_{\mathcal{M}}\mathcal{M})^{-1}. \tag{13}$$

where i_0 and $i_{\mathcal{M}}$ are assumed constants.

Remark 1. We assume in this paper that η can be a controlled parameter. This requires knowing the air flow rate passing through the propulsion and having good description of the characteristic $\Phi_i I_{\text{sp}}^{\Phi_i}(\Phi_i)$. The latter is possible since extensive testing of propulsion is realized during vehicle's conception. Using our propulsion modelling, this characteristic is, in fact, an unimodal function, which actually limits the propulsion operating range.

[2] One can remark that to simplify notation, ε include some factors.

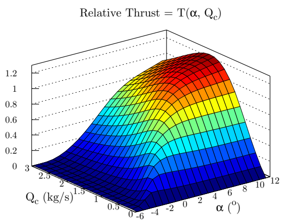

Fig. 3. Thrust force variation with respect to angle of attack and fuel flow rate.

3.4 Rigid body mechanics

We restrict ourselves to the vertical plane and assume that the mass does not vary during flight. Consequently, we can apply Newton's dynamics theory. So, by noting $\mathbf{R}_{\mathrm{A}\to 0}$ the orthogonal matrix transforming a velocity related frame to the inertial frame, we can write

$$\dot{\widehat{\mathbf{R}_{\mathrm{A}\to 0}\mathbf{V}}} = \mathbf{R}_{\mathrm{A}\to 0}\dot{\mathbf{V}} + \frac{\partial \mathbf{R}_{\mathrm{A}\to 0}}{\partial \gamma}\mathbf{V}\dot{\gamma} = \mathbf{R}_{\mathrm{A}\to 0}\frac{\mathbf{F}}{m}, \tag{14}$$

and left-multiplying (14) by $\mathbf{R}_{\mathrm{A}\to 0}^{\top}$ leads to the expression of the velocity dynamics (V, γ).

Moreover, since the dynamics modeling is restricted to the plane $(O, \boldsymbol{x}_0, \boldsymbol{z}_0)$, the rotational dynamic is straight forward. Hence the complete modelling of the vehicle is defined in vertical plane by

$$\dot{V} = \frac{1}{m}\mathbf{F}.\boldsymbol{x}_{\mathrm{A}}, \tag{15a}$$

$$\dot{\gamma} = \frac{1}{mV}\mathbf{F}.\boldsymbol{z}_{\mathrm{A}}, \tag{15b}$$

$$\dot{\alpha} = q - \dot{\gamma}, \tag{15c}$$

$$\dot{q} = \frac{M}{J}, \tag{15d}$$

where $\boldsymbol{x}_{\mathrm{A}}$ and $\boldsymbol{z}_{\mathrm{A}}$ are velocity related frame axes.

4. SHAPING UP THE PLANT IN FEEDBACK FORM

As can be seen on Figure 1, there is a strong dependency of force coefficients w.r.t. elevator angles. This technological specificity —very different from what can be observed in most aircraft models where the forces induced by control surface is neglected— is due to the presence of huge control wings necessary to counteract the momentum induced by the intrados of the vehicle. In a control perspective, such a feature is known to induce a non-minimum phase behavior; for more details, see *e.g.* Parker et al. (2007) and discussion in Menon (2001) following Wang and Stengel (2000).

Since there exists a time scale separation between rotational dynamic and velocity dynamic, we treat this problem as a singularly perturbed hierarchical control.

4.1 Singularly perturbed hierarchical control

Theorem 2. Let a singular perturbed and sufficiently smooth system be

$$\dot{x} = f(x, z, u), \tag{16a}$$

$$\epsilon\dot{z} = g(x, z, u), \tag{16b}$$

where $g(x, z, u)$ is affine in u and $u = \phi_f(z, z_c)$ is a fast x-parameterized control law stabilising exponentially $z \to z_c$ uniformly in x when $\epsilon \to 0$. Suppose that the equation $g(x, z_c, u) = 0$ have only one isolated root given by $u = \varphi(x, z_c)$ which describes the fast equilibrium manifold.

Then, there exists a positive time delay Δt and a minimum scale separation ϵ^* such that the control law $z_c = \phi_s(x)$ which stabilises the perturbed system

$$\dot{x} = \bar{f}(x, z) = f(x, z, \varphi(x, z)), \tag{17}$$

stabilises also the overall system (16) $\forall t \in [t_0 + \Delta t; \infty[$, for a sufficiently small time scale factor $\epsilon < \epsilon^*$.

It will be shown that the fast control, in the slow subsystem, can be replaced by it expression on the equilibrium manifold, which triangularises the overall system. This way, and using the above assumptions, we can stabilise the system (16) with a control law designed on the following lower triangular system

$$\dot{x} = \bar{f}(x, z), \tag{18a}$$

$$\epsilon\dot{z} = g(x, z, u). \tag{18b}$$

A backstepping procedure allows simplifying the nonlinear control design.

Proof. Since fast subsystem is affine in control, we can write

$$u = \varphi(x, z_c) + \tilde{\phi}_f(\tilde{z}), \tag{19}$$

where $\tilde{z} = z - z_c$ is the distance to the equilibrium manifold. Thus, the boundary layer equation

$$\dot{\tilde{z}} = \tilde{g}_x(z_c, \tilde{z}, \tilde{\phi}_f(\tilde{z})) \tag{20}$$

is exponentially stable by assumption. By application of Tikhonov's theorem, present in Kokotovic et al. (1986), we know that there exists ϵ^* and Δt such that under assumptions the perturbed system (18) is equivalent to the original system (16) over a time delay Δt. This complete the proof.

4.2 Application to the hypersonic vehicle

A critical step for the use of the preceding theorem is to solve the fast equilibrium manifold equation $g(x, z, u) = 0$. In aeronautics, this equation relies on the rotational dynamics equilibrium equation. Assuming we have a linear approximation of the aerodynamic momentum expression, the fast equilibrium manifold is given by

$$C_M(\alpha, \delta) = 0 = C_{M0} + \frac{\partial C_M}{\partial \alpha}\alpha + \frac{\partial C_M}{\partial \delta}\delta, \tag{21}$$

and theorem 2 allows us to consider that fins angles have to stabilise fast dynamic so quickly that its value on the equilibrium manifold is a sufficiently good approximation to use it in the slow dynamics. Therefore,we can rewrite equation (4) as

$$\bar{C}_L = C_{L_0} + C_{L_\alpha}\alpha. \tag{22}$$

As far as drag is concerned, the polar equation $\bar{C}_D \approx C_{D0} + C_{Dl}\bar{C}_L^2$ provides an elegant way to describe the drag function, since it remains valid in the hypersonic domain.

5. STABILITY AND STABILISATION OF THE (V, γ) SLOW SUBSYSTEM

Assumption 1. The curvature of the earth is neglected and Coriolis force is replaced by a constant offset on gravity.

Assumption 2. The air density $\rho(h)$ is supposed to be constant for the domain of altitude.

Assumption 3. Thrust T is assumed to be state independent.

Under the preceding assumptions, we can write the (V, γ) dynamics as

$$\dot{V} = t - d(l)\Lambda(V) - g\sin\gamma, \tag{23a}$$

$$\dot{\gamma} = \frac{l\Lambda(V) - g\cos\gamma}{V}, \tag{23b}$$

where $t = \frac{T}{m}$, $d(l) = \frac{1}{2m}\rho(h)S_{\text{ref}}\bar{C}_D$ and $l = \frac{1}{2m}\rho(h)S_{\text{ref}}\bar{C}_L$. Such a system is a variation of the Zhukovskii oscillator (see Andronov et al. (1987) and references inside).

5.1 *Existence and uniqueness of equilibrium*

Let f be the lift-to-drag ratio; the speed equilibrium is given by solving the polynomial equation

$$(l\Lambda(V))^2 + \left(t - \frac{l}{f}\Lambda(V)\right)^2 - g^2 = 0. \tag{24}$$

Since $\Lambda(V)$ maps $\mathbb{R}_+ \to \mathbb{R}_+$, we are looking for real positive roots of equation (24). Calculus of the discriminant to find a condition for real roots existence leads to the condition

$$t^2 < g^2\left(1 + \frac{1}{f^2}\right). \tag{25}$$

Moreover there exists only one positive solution if $t^2 < g^2$. Else, in the case

$$g^2 < t^2 < g^2\left(1 + \frac{1}{f^2}\right), \tag{26}$$

there exists another (low-speed and high flight path angle) equilibrium position where flight is mainly sustained by thrust. However, this equilibrium is not in the foreground of this study, since hypersonic atmospheric flight cannot be maintained with such high flight path angle.

Assumption 4. The hypersonic airbreathing vehicle is supposed to keep the flight path angle low, *e.g.* γ is small.

This last assumption simplifies (V, γ) dynamics as

$$\dot{V} = t - d(l)\Lambda(V) - g\gamma, \tag{27a}$$

$$\dot{\gamma} = \frac{l\Lambda(V) - g}{V}, \tag{27b}$$

and an approximation of the equilibrium is given by

$$\gamma_0 = \frac{t_0}{g} - \frac{1}{f_0}, \qquad V_0 = \Lambda^{-1}\left(\frac{g}{l_0}\right). \tag{28}$$

5.2 *Stability of the equilibrium*

Definition 1. ($\mathcal{C}^1$-dissipativeness). The system $\dot{x} = \zeta(x)$ is $\mathcal{C}^1$-dissipative if there exists a $\mathcal{C}^1$ Lyapunov function W (*i.e.* positive definite and proper) satisfying

$$\frac{\partial W}{\partial x}(x)\zeta(x) \leq 0 \quad \forall x \in \mathbb{R}^n.$$

Proposition 3. (Dissipativeness of flight). Given three positive constants l_0, d_0 and $t_0 < g$, the Zhukovskii oscillator defined by (27) is $\mathcal{C}^1$-dissipative and its unique equilibrium is exponentially stable.

Proof. Uniqueness of equilibrium is already established, so we focus on stability. Choosing as a Lyapunov function

$$W(V, \gamma) = \int_{\Lambda^{-1}(\frac{g}{l_0})}^{V} \frac{\Lambda(v) - \frac{g}{l_0}}{v} dv + \frac{g}{2l_0}\left(\gamma - \frac{t_0}{g} + \frac{d_0}{l_0}\right)^2, \tag{29}$$

with

$$\frac{\partial W}{\partial V} = \frac{\Lambda(V) - \frac{g}{l_0}}{V}, \quad \frac{\partial W}{\partial \gamma} = \frac{g}{l_0}\left(\gamma - \frac{t_0}{g} + \frac{d_0}{l_0}\right), \tag{30}$$

leads to

$$\dot{W}(V, \gamma) = -\frac{d_0}{V}\left(\Lambda(V) - \Lambda(V_0)\right)^2. \tag{31}$$

This proves the $\mathcal{C}^1$-dissipativeness of (27). Secondly, given a compact set C included in a neighborhood of the equilibrium point, the set $\{(V, \gamma) \in C : \dot{W}(V, \gamma) = 0\}$ contains a unique invariant point (V_0, γ_0). Therefore, using LaSalle theorem we can establish the asymptotic stability of (V_0, γ_0).

Exponential stability can be proved by studying the equilibrium tangent's approximation: under the condition [3]

$$\frac{\partial \Lambda}{\partial V} < 4f^2\frac{\Lambda(V_0)}{V_0}, \tag{32}$$

the real part of the eigenvalues of the dynamic first order approximation is given by $-\frac{l_0}{2f}\frac{\partial \Lambda}{\partial V}$. This ends the proof.

5.3 *Stabilisation of the (V, γ) subsystem*

The reference trajectory is assumed to be given in terms of a sequence of (V_c, γ_c) to be reached. Thus we can find using (28) a couple l_c and t_c, such that equilibrium of the system (27) coincides with (V_c, γ_c).

From the $\mathcal{C}^1$-dissipativeness of the system, it appears that the stabilisation of the system can be achieved by any control law of the form $u = \varphi\left(-L_g W(V, \gamma)\right) + u_c$, where φ is a monotonic increasing $\mathcal{C}^0$ function such that $\forall x \in \mathbb{R}, \varphi(x)x \geq 0$. This allows using bounded functions as control laws.

For example, defining $\text{sat}(x, a, b) = \min(\max(x, a), b)$; a couple of control laws for l and t could be

$$l = \text{sat}\left(-k_l\frac{\partial W}{\partial \gamma}V + l_c, \underline{l}, \bar{l}\right), \tag{33}$$

$$t = \text{sat}\left(-k_t\frac{\partial W}{\partial V} + t_0, \underline{t}, \bar{t}\right) + (d(l) - d(l_c))\Lambda(V), \tag{34}$$

which can be bounded if we restrict $\Lambda(V)$ to a closed set. The Figure 4 shows the phase plane of a such controlled (V, γ) subsystem.

5.4 *Extension to the airbreathing case*

We no longuer take into account assumption 3. In equation (11) thrust was written as function of α and V. But from (22) a certainty equivalence exists between α and l. So we can rewrite thrust as

$$t = \varepsilon(l)I_{\text{sp}}^V(V)\eta, \tag{35}$$

where ε is a saturated function, locally affine in l, obtained by replacing α in (12). The control inputs are now η and l; so the (V, γ) subsystem has to be rewritten as

[3] Verified in practice; when $\Lambda(V) = V^2$ it reduce to $f^2 > \frac{1}{2}$.

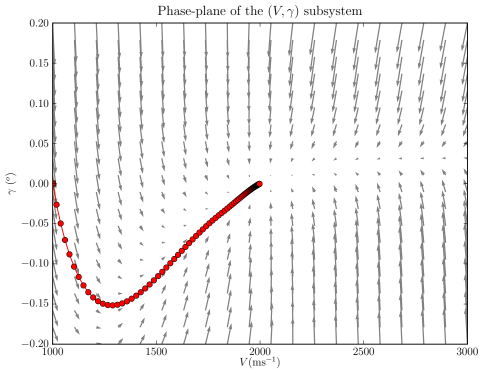

Fig. 4. Phase plane of the controlled (V,γ) subsystem, with reference $(V_c,\gamma_c)=(2000,0)$, saturations $(\bar{l},\underline{l})=(-\frac{1}{2}l_c, 2l_c)$ and $(\bar{t},\underline{t})=(0,2t_c)$, and gains $(k_l,k_t)=(10^{-8},4.10^{-3})$.

$$\dot{V}=\varepsilon(l)I_{\mathrm{sp}}^V(V)\eta-d(l)\Lambda(V)-g\gamma, \tag{36a}$$
$$\dot{\gamma}=\frac{l\Lambda(V)-g}{V}, \tag{36b}$$

for which exists an equilibrium at

$$V_0=\Lambda^{-1}\left(\frac{g}{l_0}\right),\qquad \gamma_0=\frac{\varepsilon(l_0)I_{\mathrm{sp}}^V(V_0)\eta_0}{g}-\frac{1}{f_0}. \tag{37}$$

Proposition 4. ($\mathcal{C}^1$-dissipativeness of airbreathing flight). Given three positive constants $l_0:\varepsilon(l_0)>0$, d_0 and $\eta_0: t(l_0,V_0)<g$, the airbreathing flight defined by (36) is $\mathcal{C}^1$-dissipative and its equilibrium, defined by (37), is asymptotically stable.

Proof. Derivating Lyapunov function (29) leads to

$$\begin{aligned}\dot{W}(V,\gamma)=&-\frac{d_0}{V}\left(\Lambda(V)-\Lambda(V_0)\right)^2\\&+\frac{\varepsilon(l_0)\eta_0}{V}\left(\Lambda(V)-\Lambda(V_0)\right)\left(I_{\mathrm{sp}}^V(V)-I_{\mathrm{sp}}^V(V_0)\right).\end{aligned} \tag{38}$$

Since $\varepsilon(l_0)\eta_0$ is positive, and Λ and I_{sp}^V are monotonic functions respectively increasing and decreasing, we conclude to the $\mathcal{C}^1$-dissipativeness of system (36). Asymptotic stability can be established similarly as in proposition 3.

Due to the bilinear input effect, controlling system (36) leads to a control allocation problem between thrust and drag effect. Solving such a nonlinear problem —*e.g.* by cost formulation— is a difficult problem which will not be addressed here. In a perspective of extending control law (34) we will keep the same l as in Section 5.3. By noting l_c and η_c the couple of constant controls such that equilibrium of the (V,γ) subsystem coincides with (V_c,γ_c), a stabilising control law for the system (36) is given by

$$l=\mathrm{sat}\left(-k_l\frac{\partial W}{\partial\gamma}V+l_c,\underline{l},\bar{l}\right), \tag{39}$$
$$\begin{aligned}\eta=\eta_c+\frac{1}{\varepsilon(l)I_{\mathrm{sp}}^V(V)}\Big\{&\mathrm{sat}\left(-k_t\frac{\partial W}{\partial V}\varepsilon(l)I_{\mathrm{sp}}^V(V),\underline{t},\bar{t}\right)\\&+(d(l)-d(l_c))\Lambda(V)-\eta_c I_{\mathrm{sp}}^V(V_c)\big(\varepsilon(l)-\varepsilon(l_c)\big)\Big\},\end{aligned} \tag{40}$$

which can be bounded if we restrict V to a closed set.

6. BACKSTEPPING OF THE LIFT CONTROL LAW

As the slow subsystem control law has been designed, backstepping is a powerful tool to find a control Lyapunov function stabilising the overall system by extending (29). We need two back steps in order to stabilize lift and pitch dynamics.

6.1 First step: stabilisation of α

We extend the dynamic of the system to

$$\dot{V}=\varepsilon(l)I_{\mathrm{sp}}^V(V)\eta-d(l)\Lambda(V)-g\gamma, \tag{41a}$$
$$\dot{\gamma}=\frac{l\Lambda(V)-g}{V}, \tag{41b}$$
$$\dot{l}=C_{L_\alpha}\left(q-\dot{\gamma}\right). \tag{41c}$$

Let $\phi_\gamma(\gamma)$ be the control law defined in (39), the Lyapunov function [4]

$$W_2(V,\gamma,l)=k_\epsilon W(V,\gamma,l)+\frac{1}{2}\left(l-\phi_\gamma(\gamma)\right)^2 \tag{42}$$

is derived along the solutions as

$$\begin{aligned}\dot{W}_2(V,\gamma,l)=&\,k_\epsilon\dot{W}(V,\gamma,\phi_\gamma(\gamma))-C_{L_\alpha}\frac{\Lambda(V)}{V}\left(l-\phi_\gamma(\gamma)\right)^2\\&+\left(l-\phi_\gamma(\gamma)\right)\Big[k_\epsilon\frac{\partial W}{\partial\gamma}\frac{\Lambda(V)}{V}\\&+C_{L_\alpha}\left(q-\frac{\phi_\gamma(\gamma)\Lambda(V)-g}{V}\right)-\dot{\widehat{\phi_\gamma(\gamma)}}\Big],\end{aligned} \tag{43}$$

and therefore is definite negative by choosing

$$q=\mathrm{sat}(-k_l(l-\phi_\gamma(\gamma)),\underline{q},\bar{q})+\mathcal{Q}, \tag{44}$$

where

$$\mathcal{Q}=C_{L_\alpha}^{-1}\left(\dot{\widehat{\phi_\gamma(\gamma)}}-k_\epsilon\frac{\partial W}{\partial\gamma}\frac{\Lambda(V)}{V}+C_{L_\alpha}\frac{\phi_\gamma(\gamma)\Lambda(V)-g}{V}\right). \tag{45}$$

6.2 Second step: stabilisation of q

As equation (44) is defined as a q control law, then we consider

$$\dot{V}=\varepsilon(l)I_{\mathrm{sp}}^V(V)\eta-d(l)\Lambda(V)-g\gamma, \tag{46a}$$
$$\dot{\gamma}=\frac{l\Lambda(V)-g}{V}, \tag{46b}$$
$$\dot{l}=C_{L_\alpha}\left(q-\dot{\gamma}\right), \tag{46c}$$
$$\dot{q}=c_0+c_l l-c_q q+c_\delta(1+c_{\delta l}l)\delta. \tag{46d}$$

Noting $\phi_l(l)$ the control law defined in (44), the Lyapunov function

$$W_3(V,\gamma,l,q)=W_2(V,\gamma,l,\phi_l(l))+\frac{1}{2}(q-\phi_l(l))^2 \tag{47}$$

is derived along the solution as

$$\begin{aligned}\dot{W}_3(V,\gamma,l,q)=&\,\dot{W}_2(V,\gamma,l,\phi_l(l))-c_q(q-\phi_l(l))^2\\&+(q-\phi_l(l))\Big[C_{L_\alpha}(l-\phi_\gamma(\gamma))+c_0+c_l l\\&-c_q\phi_l(l)+c_\delta(1+c_{\delta l}l)\delta-\dot{\widehat{\phi_l(l)}}\Big].\end{aligned} \tag{48}$$

Finally, choosing the control law

$$\delta=\mathrm{sat}(-k_q(q-\phi_l(l))(1+c_{\delta l}l),\underline{\delta},\bar{\delta})+\mathcal{D} \tag{49}$$

[4] The k_ϵ coefficient is needed because of time scale separation.

globally stabilizes the hypersonic vehicle's model defined by (46), where

$$\mathcal{D} = -(c_\delta(1 + c_{\delta l} l))^{-1} \cdot \left(C_{L_\alpha}(l - \phi_\gamma(\gamma)) + c_0 + c_l l - c_q \phi_l(l) - \dot{\widehat{\phi_l(l)}} \right). \quad (50)$$

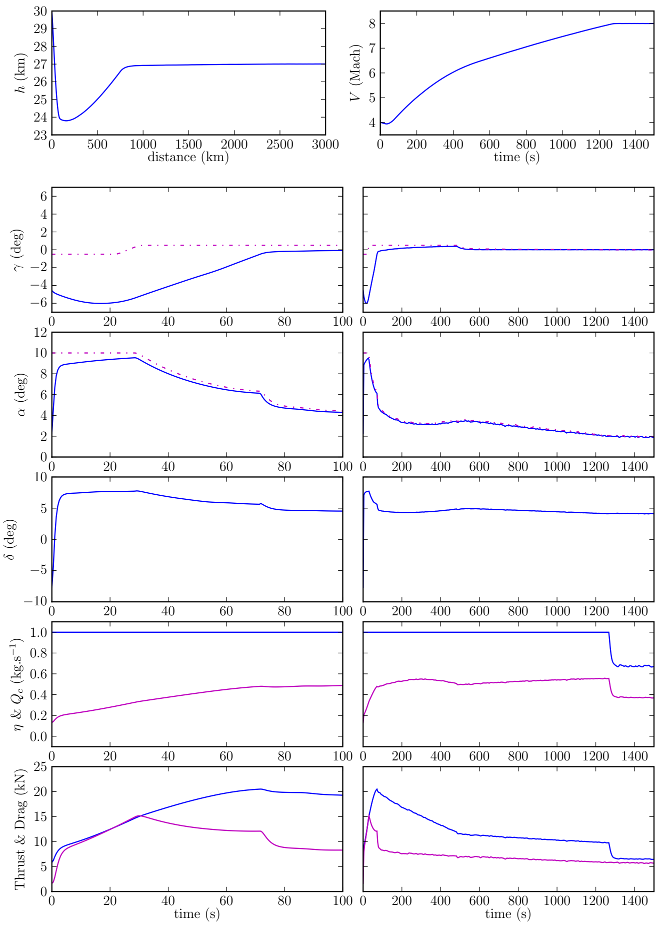

Fig. 5. Stabilisation of a representative nonlinear vehicle using the control law (49).

7. SIMULATION RESULTS

The simulation results of the control law (49)[5] are presented on Figure 5. We used for simulation a fully nonlinearly parameterised aerodynamics and an altitude-dependent perturbation wind. In order to illustrate some limits of the controlled HSV, initialisation has been set far from equilibrium.

As it can be seen, the influence of propulsion on rotational dynamic is clearly covered by aerodynamic forces, and robustness to parameters uncertainties[6] is satisfied.

The control law has been shaped in order to saturate the AoA of the vehicle: first it is saturated by a angle limit (*e.g.* to guarantee air inlet properties), and next, after altitude decreases, AoA is saturated by vertical load factor (*e.g.* to preserve body's structure).

[5] We only shaped γ_c in order to stabilize altitude.

[6] Variation of $\pm 20\%$ on aerodynamics and propulsion parameters.

As far as propulsion is concerned, we inversed η considering linear approximation of the characteristic $\Phi_i I_{\text{sp}}^{\Phi_i}(\Phi_i)$. The main part of propulsion available is currently used during acceleration of the HSV, and the control allocation problem discussed in Section 5.4 appears slightly during high AoA phases, since thrust remain available even in this situation where drag dramatically increase.

8. CONCLUSION

The global nonlinear control law which has been addressed here is a first attempt to assign a Lyapunov based flight nonlinear control law, allowing to trade with hard nonlinearities like saturations. The assumptions made here are not very restrictive. Extension of such a control law can be performed on any flying vehicle with little small path angle.

There is many perspectives opened by such a nonlinear control law. We have to extend our work to the fully dimensioned model, and to take in account of altitude impact on the dynamics. Others perspectives are to trade with the control allocation problem. We also need to robustify this control law to model uncertainties, using *e.g.* nonlinear integrators. Thirdly, we have to find guarantees of boundedness of η without obtaining suboptimal gains on k_t. Finally we need to find robust control laws guaranteeing stability under saturated angle of elevators δ.

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