Nonlinear Control of a Airbreathing Hypersonic Vehicle:
A Backstepping Approach

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Abstract: Nowadays, airbreathing vehicle’s control represents a very motivating challenge, as it includes the respect of some physical constraints, hard nonlinearities in thrust operation, and also a non minimum phase behavior due to fins’ size. In this paper we describe methods for simultaneous guidance and control nonlinear laws design relying on Lyapunov functions theory. Some simulation are presented, based on a realistic modelling of such a vehicle.

Keywords: Nonlinear control, Flight control, Lyapunov function, Hypersonic waveriders.

1. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>ρ</td>
<td>Air density</td>
<td>kg/m³</td>
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<tr>
<td>m</td>
<td>Body mass</td>
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<tr>
<td>g₀</td>
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<td>J</td>
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<tr>
<td>M</td>
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<td>Reference length</td>
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<td>fₛ</td>
<td>Stoichiometric factor</td>
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</tr>
<tr>
<td>Sᵣ</td>
<td>Wing area</td>
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<tr>
<td>h</td>
<td>Altitude</td>
<td>m</td>
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<tr>
<td>γ</td>
<td>Flight path angle</td>
<td>rad</td>
</tr>
<tr>
<td>α</td>
<td>Angle of attack (AoA)</td>
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<tr>
<td>q</td>
<td>Pitch rate</td>
<td>rad.s⁻¹</td>
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<tr>
<td>δ</td>
<td>Angle of elevators</td>
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<tr>
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<td>Drag factor</td>
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<td>Pressure factor w.r.t. V</td>
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<tr>
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<td>Lift factor</td>
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<td>Cₗ</td>
<td>Reduced lift coefficient</td>
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<tr>
<td>C₉</td>
<td>Reduced drag coefficient</td>
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<td>f</td>
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<td>η</td>
<td>Thrust ratio</td>
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2. INTRODUCTION

The various interests of airbreathing propulsion had been identified for a long time in various applications, see e.g. Lentsch et al. (2003) which resume airbreathing vehicle’s development in ONERA, or Falempin and Serre (2008) for a description of LEA project status. As an example, in the missile domain, hypersonic cruise at high altitude tends to insure long dynamic range and decreases interception’s risks. In the case of spacecraft applications, the ability to use air-picked oxydizer on a major part of the trajectory could strongly increase the mass ratio to orbital speed (currently oxydizer is close to 75% of the mass of a launcher like Ariane V).

Controlling such a vehicle has been addressed in several ways, relying on linear control theory (see e.g. Sigthorsson et al. (2008) and references inside), dynamic inversion, and sliding-modes control (see Wang and Stengel (2000) and Xu and Ioannou (2004)). However, most of these works did not considered hard nonlinearities induced by propulsion system and non minimum phase behavior.

Our work aims at providing a model of such a vehicle, and then studying stability and nonlinear stabilisation by control Lyapunov function along a trajectory described in the vertical plane. Lyapunov theory allows us using the full range of our different actuators, taking in account potential saturations. It is also known to insure certain degree of robustness and optimality properties.

The paper is organized as follow. We firstly describe in Sections 3 and 4 the model structure and the different assumptions made on the modelling structure of the aerodynamics and thrust. Our aerodynamic structure is designed from semi-empiric methods, and the resulting model structure similar to what has been published in Parker et al. (2007). We will next assume in Section 5 that a reference trajectory exists in terms of flight speed and path angle. We aim at designing in Section 6 a closed-loop guidance and control law that will stabilize the vehicle along this trajectory. In this paper, focuses will be put on specificities brought by these waverider concepts, compared with more traditional aircrafts. Finally, examples will be presented in Section 7 to illustrate the performances of the approach.
3. MODELISATION OF THE VEHICLE’S DYNAMIC

3.1 Aerodynamic forces

Aerodynamic forces are usually modelled as a product of dynamic pressure—which depends on relative air velocity and air density—and a coefficient term reflecting wetted surface of the wing. This coefficient depends on wing’s geometry and relative air flow direction:

\[ F_x = \frac{1}{2} \rho V^2 S_{ref} C_x, \]  

where \( C_x \) is an aerodynamic coefficient. It varies with angle of attack, angle of elevators and Mach number; as Figure 1 illustrates, it is well known in the hypersonic domain that the \( C_x \) coefficients decrease w.r.t. Mach number. Here we rather use an aerodynamic force model like

\[ F_x = \frac{1}{2} \rho V^2 \Lambda(V) C_x(\alpha, \delta), \]  

where \( \Lambda(V) \) is a monotonic increasing function relying on flow pressure\(^1\). We will use

\[ \Lambda(V) = \frac{V^2}{s_0 + s_1 V}. \]  

This kind of modelling is, in fact, motivated by analogy with hypersonic Newtonian approximation, and it will be assumed consequently a Mach-independent lift-to-drag ratio. This property will be exploited by the control presented in the following.

We finally modelled our forces coefficients as a linearly parameterized function in \((\alpha, \delta)\) using data obtained from semi-empiric methods (see Figure 1 and 2):

\[ C_L = \begin{bmatrix} L_L(1 \times \delta) \end{bmatrix}^T, \quad C_D = \begin{bmatrix} L_D(1 \times \delta) \end{bmatrix}^T + \begin{bmatrix} \alpha \times \delta \end{bmatrix} \begin{bmatrix} Q_D(\alpha \times \delta) \end{bmatrix}^T. \]  

3.2 Aerodynamic momentum

Similarly to the aerodynamics forces, aerodynamic momentum is usually modelled as

\[ M_x = \frac{1}{2} \rho V^2 S_{ref} l_{ref} C_x. \]  

\(^1\) In the case of a subsonic aircraft, we can simply get \( \Lambda(V) = V^2 \).

Fig. 2. Aerodynamic moment coefficient with respect to angle of attack, angle of elevators, and Mach number. As it can be seen on Figure 2, aerodynamic pitch coefficient is roughly independent from Mach, and varies quasi-linearly with angle of attack and angle of elevators. There exists also a damping effect to be added, assumed to be linear in \( \frac{\rho V}{m} q \). So we modelled the momentum coefficient as the linearly parameterized function:

\[ C_M = C_{M0} + C_{M\delta} \alpha + C_{M\delta} \frac{\rho V}{m} q + (C_{M\delta \alpha} + C_{M\delta \alpha}) \delta. \]  

3.3 Scramjet thrust

Scramjet propulsion is a very complex phenomenon which will be modelled using a specific impulse function depending on Mach number and air-fuel ratio:

\[ T = g_0 Q_c I_{sp}(\alpha, \delta), \]  

Figure 3 illustrates the input/output behavior of such a model. It appears that specific impulse can itself be split as a product of two terms:

- the dependency on Mach —relying on efficiency—monotonic and decreasing in all hypersonic domain, which can be assimilated to a dependency in \( V \);
- the dependency on air-fuel ratio, an unimodal function passing through a maximum near \( \Phi_i = 0.7 \).

As

\[ Q_c = \Phi_i f_s Q_A = f_s \rho V \varepsilon(\alpha, \delta), \]  

we can rewrite equation (8) as

\[ T = g_0 f_s \rho \varepsilon(\alpha, \delta, \Phi_i) I_{sp}(\Phi_i) V I_{sp}(V), \]  

but since the Scramjet is usually designed to keep the term \( \rho \varepsilon(\alpha_0, \delta, \Phi_i) \) constant, we thus can model thrust as\(^2\)

\[ T = m e(\Phi_i) I_{sp}(V) \eta, \quad \eta \in [0; 1] \]  

where \( \eta \propto \Phi_i I_{sp}(\Phi_i) \) is a bounded propulsion characteristic which is assumed to be locally invertible. Finally we modelled air-captation model as a locally affine function

\[ \varepsilon(\alpha) = \max(\varepsilon_0, \min(\varepsilon_0 + \varepsilon_{10} \alpha, \varepsilon_{10} + \varepsilon_{10} \alpha)), \]  

and thrust efficiency w.r.t. Mach as

\[ I_{sp}(V) = (i_0 + i_{\delta} M)^{-1}. \]  

\(^2\) One can remark that to simplify notation, \( \varepsilon \) include some factors.
Let a singular perturbed and sufficiently smooth system be
\[
\dot{x} = f(x, z, u), \quad (16a)
\]
\[
\epsilon \dot{z} = g(x, z, u), \quad (16b)
\]
where \(g(x, z, u)\) is affine in \(u\) and \(u = \phi_f(z, z_c)\) is a fast \(x\)-parameterized control law stabilising exponentially \(z \to z_c\) uniformly in \(x\) when \(\epsilon \to 0\). Suppose that the equation \(g(x, z_c, u) = 0\) have only one isolated root given by \(u = \varphi(x, z_c)\) which describes the fast equilibrium manifold.

Then, there exists a positive time delay \(\Delta t\) and a minimum scale separation \(\epsilon^*\) such that the control law \(z_c = \phi_s(x)\) which stabilises the perturbed system
\[
\dot{x} = f(x, z, \varphi(x, z)), \quad (17)
\]
stabilises also the overall system (16) \(\forall t \in [t_0 + \Delta t; \infty]\), for a sufficiently small time scale factor \(\epsilon < \epsilon^*\).

It will be shown that the fast control, in the slow subsystem, can be replaced by it expression on the equilibrium manifold, which triangularises the overall system. This way, and using the above assumptions, we can stabilise the system (16) with a control law designed on the following lower triangular system
\[
\dot{x} = \bar{f}(x, z), \quad (18a)
\]
\[
\epsilon \dot{z} = \bar{g}(x, z, u). \quad (18b)
\]
A backstepping procedure allows simplifying the nonlinear control design.

**Proof.** Since fast subsystem is affine in control, we can write
\[
u = \varphi(x, z_c) + \tilde{\phi}_f(\tilde{z}), \quad (19)
\]
where \(\tilde{z} = z - z_c\) is the distance to the equilibrium manifold. Thus, the boundary layer equation
\[
\dot{\tilde{z}} = \tilde{g}_u(z_c, \tilde{z}, \tilde{\phi}_f(\tilde{z})), \quad (20)
\]
is exponentially stable by assumption. By application of Tikhonov’s theorem, present in Kokotovic et al. (1986), we know that there exists \(\epsilon^*\) and \(\Delta t\) such that under assumptions the perturbed system (18) is equivalent to the original system (16) over a time delay \(\Delta t\). This complete the proof.

### 4.2 Application to the hypersonic vehicle

A critical step for the use of the preceding theorem is to solve the fast equilibrium manifold equation \(g(x, z, u) = 0\). In aeronautics, this equation relies on the rotational dynamics equilibrium equation. Assuming we have a linear approximation of the aerodynamic momentum expression, the fast equilibrium manifold is given by
\[
C_M(\alpha, \delta) = 0 = C_M \alpha + \frac{\partial C_M}{\partial \alpha} + \frac{\partial C_M}{\partial \delta}, \quad (21)
\]
and theorem 2 allows us to consider that fins angles have to stabilise fast dynamic so quickly that its value on the equilibrium manifold is a sufficiently good approximation to use it in the slow dynamics. Therefore, we can rewrite equation (4) as
\[
\dot{C}_L = C_{L_\alpha} + C_{L_\delta} \alpha. \quad (22)
\]
As far as drag is concerned, the polar equation \(\dot{C}_D \approx C_{D_0} + C_{D_\alpha} \dot{C}_L^2\) provides an elegant way to describe the drag function, since it remains valid in the hypersonic domain.

### 5. STABILITY AND STABILISATION OF THE \((V, \gamma)\) SLOW SUBSYSTEM

**Assumption 1.** The curvature of the earth is neglected and Coriolis force is replaced by a constant offset on gravity.
Assumption 2. The air density $\rho(h)$ is supposed to be constant for the domain of altitude.

Assumption 3. Thrust $T$ is assumed to be state independent.

Under the preceding assumptions, we can write the $(V, \gamma)$ dynamics as

\[
\dot{V} = t - d(l) \Lambda(V) - g \sin \gamma, \quad (23a) \\
\dot{\gamma} = \frac{\Lambda(V) - g \cos \gamma}{V}, \quad (23b)
\]

where $t = \frac{T}{m}$, $d(l) = \frac{\rho(h)}{2m} S_{ref} C_D$ and $l = \frac{1}{\rho(h)} S_{ref} C_L$. Such a system is a variation of the Zhukovskii oscillator (see Andronov et al. (1987) and references inside).

5.1 Existence and uniqueness of equilibrium

Let $f$ be the lift-to-drag ratio; the speed equilibrium is given by solving the polynomial equation

\[
(\Lambda(V))^2 + \left(t - \frac{1}{f} \Lambda(V)\right)^2 - g^2 = 0. \quad (24)
\]

Since $\Lambda(V)$ maps $\mathbb{R}_+ \to \mathbb{R}_+$, we are looking for real positive roots of equation (24). Calculus of the discriminant to find a condition for real roots existence leads to the condition

\[
t^2 < g^2 \left(1 + \frac{1}{f^2}\right). \quad (25)
\]

Moreover there exists only one positive solution if $t^2 < g^2$. Else, in the case

\[
g^2 < t^2 < g^2 \left(1 + \frac{1}{f^2}\right), \quad (26)
\]

there exists another (low-speed and high flight path angle) equilibrium position where flight is mainly sustained by thrust. However, this equilibrium is not in the foreground of this study, since hypersonic atmospheric flight cannot be maintained with such high flight path angle.

Assumption 4. The hypersonic airbreathing vehicle is supposed to keep the flight path angle low, e.g. $\gamma$ is small.

This last assumption simplifies $(V, \gamma)$ dynamics as

\[
\dot{V} = t - d(l) \Lambda(V) - g \gamma, \quad (27a) \\
\dot{\gamma} = \frac{\Lambda(V) - g}{V}, \quad (27b)
\]

and an approximation of the equilibrium is given by

\[
\gamma_0 = \frac{t_0}{g} - \frac{1}{f_0}, \quad V_0 = \Lambda^{-1} \left(\frac{g}{t_0}\right). \quad (28)
\]

5.2 Stability of the equilibrium

Definition 1. ($C^1$-dissipativeness). The system $\dot{x} = \zeta(x)$ is $C^1$-dissipative if there exists a $C^1$ Lyapunov function $W$ (i.e. positive definite and proper) satisfying

\[
\frac{\partial W}{\partial x}(x) \zeta(x) \leq 0 \quad \forall x \in \mathbb{R}^n.
\]

Proposition 3. (Dissipativeness of flight). Given three positive constants $t_0$, $d_0$ and $t_0 < g$, the Zhukovskii oscillator defined by (27) is $C^1$-dissipative and its unique equilibrium is exponentially stable.

**Proof.** Uniqueness of equilibrium is already established, so we focus on stability. Choosing as a Lyapunov function

\[
W(V, \gamma) = \int_{\Lambda^{-1}(\frac{g}{t_0})}^V \left(\frac{\Lambda(x) - \frac{g}{t_0}}{\gamma - \frac{t_0}{g} + \frac{d_0}{t_0}}\right)^2 dx, \quad (29)
\]

with

\[
\frac{\partial W}{\partial V} = \frac{\Lambda(V) - \frac{g}{t_0}}{V}, \quad \frac{\partial W}{\partial \gamma} = \frac{g}{t_0} \left(\gamma - \frac{t_0}{g} + \frac{d_0}{t_0}\right), \quad (30)
\]

leads to

\[
\dot{W}(V, \gamma) = -\frac{d_0}{V} \left(\Lambda(V) - \Lambda(V_0)\right)^2. \quad (31)
\]

This proves the $C^1$-dissipativeness of (27). Secondly, given a compact set $C$ included in a neighborhood of the equilibrium point, the set $\{(V, \gamma) \in C : \dot{W}(V, \gamma) = 0\}$ contains a unique invariant point $(V_0, \gamma_0)$. Therefore, using LaSalle we can establish the asymptotic stability of $(V_0, \gamma_0)$.

Exponential stability can be proved by studying the equilibrium tangent’s approximation: under the condition

\[
\frac{\partial \Lambda}{\partial V} < 4f^2 \frac{\Lambda(V_0)}{V_0}, \quad (32)
\]

the real part of the eigenvalues of the dynamic first order approximation is given by $-\frac{\partial \Lambda}{\partial V}$.

This ends the proof.

5.3 Stabilisation of the $(V, \gamma)$ subsystem

The reference trajectory is assumed to be given in terms of a sequence of $(V_c, \gamma_c)$ to be reached. Thus we can find using (28) a couple $l_c$ and $t_c$ such that equilibrium of the system (27) coincides with $(V_c, \gamma_c)$.

From the $C^1$-dissipativeness of the system, it appears that the stabilisation of the system can be achieved by any control law of the form $u = \varphi(-LgW(V, \gamma)) + u_c$, where $\varphi$ is a monotonic increasing $C^0$ function such that $\forall x \in \mathbb{R}, \varphi(x)x \geq 0$. This allows using bounded functions as control laws.

For example, defining $\text{sat}(x, a, b) = \min(\max(x, a), b)$; a couple of control laws for $l$ and $t$ could be

\[
l = \text{sat} \left(-k_l \frac{\partial W}{\partial \gamma} V + l_c, \frac{1}{2} \bar{f}\right), \quad (33)
\]

\[
t = \text{sat} \left(-k_t \frac{\partial W}{\partial \gamma} V + t_0, \frac{1}{2} \bar{f}\right) + (d(l) - d(l_0)) \Lambda(V), \quad (34)
\]

which can be bounded if we restrict $\Lambda(V)$ to a closed set. The Figure 4 shows the phase plane of a such controlled $(V, \gamma)$ subsystem.

5.4 Extension to the airbreathing case

We no longer take into account assumption 3. In equation (11) thrust was written as function of $\alpha$ and $V$. But from (22) a certainty equivalence exists between $l$ and $t$. So we can rewrite thrust as

\[
t = \varepsilon(l) I_{dp}^e(V) \eta, \quad (35)
\]

where $\varepsilon$ is a saturated function, locally affine in $l$, obtained by replacing $\alpha$ in (12). The control inputs are now $\eta$ and $l$; so the $(V, \gamma)$ subsystem has to be rewritten as

\[3 \text{ Verified in practice; when } \Lambda(V) = V^2 \text{ it reduce to } f^2 > \frac{1}{2}.\]
Proof. Asymptotically stable. Due to the bilinear input effect, controlling system (36) stability can be established similarly as in proposition 3. Noting \( l \) cost formulation— is a difficult problem which will not conclude to the \( C^1 \) dissipativeness and its equilibrium, defined by (37), is asymptotically stable.

Proposition 4. \( C^1 \)-dissipativeness of airbreathing flight. Given three positive constants \( l_0 > 0, d_0 \) and \( \eta_0 \); \( l(l_0, V_0) < g \), the airbreathing flight defined by (36) is \( C^1 \)-dissipative and its equilibrium, defined by (37), is asymptotically stable.

\[ V_0 = \Lambda^{-1} \left( \frac{g}{l_0} \right), \quad \gamma_0 = \frac{\varepsilon(l_0)I_{sp}(V_0)\eta_0}{g} - \frac{1}{l_0}. \]

6. BACKSTEPPING OF THE LIFT CONTROL LAW

As the slow subsystem control law has been designed, backstepping is a powerful tool to find a control Lyapunov function stabilising the overall system by extending (29). We need two back steps in order to stabilize lift and pitch dynamics.

6.1 First step: stabilisation of \( \alpha \)

We extend the dynamic of the system to

\[
\begin{align*}
\dot{V} &= \varepsilon(l)I_{sp}(V)\eta - d(l)\Lambda(V) - g\gamma, \quad (41a) \\
\dot{\gamma} &= \frac{\Lambda(V) - g}{V}, \quad (41b) \\
\dot{l} &= C_{L_{\alpha}} (q - \dot{\gamma}). \quad (41c)
\end{align*}
\]

Let \( \phi_\alpha(\gamma) \) be the control law defined in (39), the Lyapunov function \(^4\)

\[ W_2(V, \gamma, l) = k_I W(V, \gamma, l) + \frac{1}{2} (l - \phi_\alpha(\gamma))^2 \]

derived along the solutions as

\[ W_2(V, \gamma, l) = k_I W(V, \gamma, \phi_\alpha(\gamma)) - C_{L_{\alpha}} \Lambda(V) (l - \phi_\alpha(\gamma))^2 \]

\[ + (l - \phi_\alpha(\gamma)) \left[ k_c \frac{\partial W(\Lambda(V) - g)}{\partial \gamma} \right] \]

and therefore is definite negative by choosing

\[ q = \text{sat}(-k_l(l - \phi_\alpha(\gamma), q, \dot{q}) + Q, \quad (44) \]

where

\[ Q = C_{L_{\alpha}}^{-1} \left( \phi_\alpha(\gamma) - k_c \frac{\partial W(\Lambda(V) - g)}{\partial \gamma} + C_{L_{\alpha}} \phi_\alpha(\gamma) \Lambda(V) - g \right). \quad (45) \]

6.2 Second step: stabilisation of \( q \)

As equation (44) is defined as a \( q \) control law, then we consider

\[
\begin{align*}
\dot{V} &= \varepsilon(l)I_{sp}(V)\eta - d(l)\Lambda(V) - g\gamma, \quad (46a) \\
\dot{\gamma} &= \frac{\Lambda(V) - g}{V}, \quad (46b) \\
\dot{l} &= C_{L_{\alpha}} (q - \dot{\gamma}), \quad (46c) \\
\dot{q} &= c_0 + c_1 l - c_4 q + c_5(1 + c_4 l)\delta. \quad (46d)
\end{align*}
\]

Noting \( \phi_\alpha(l) \) the control law defined in (44), the Lyapunov function

\[ W_3(V, \gamma, l, q) = W_2(V, \gamma, l, \phi_\alpha(l)) + \frac{1}{2} (q - \phi_\alpha(l))^2 \]

derived along the solution as

\[ W_3(V, \gamma, l, q) = W_2(V, \gamma, l, \phi_\alpha(l)) - c_4(q - \phi_\alpha(l))^2 \]

\[ + (q - \phi_\alpha(l)) \left[ C_{L_{\alpha}} (l - \phi_\alpha(l)) + c_0 + c_1 l \right] \]

\[ - c_4 \phi_\alpha(l) + c_5(1 + c_4 l)\delta - \phi_\alpha(l). \quad (48) \]

Finally, choosing the control law

\[ \delta = \text{sat}(-(k_q(q - \phi_\alpha(l))(1 + c_4 l), \dot{\gamma}, \dot{\delta}) + D \] (49)}

\(^4\) The \( k_c \) coefficient is needed because of time scale separation.
globally stabilizes the hypersonic vehicle’s model defined by (46), where
\[
 \mathbf{D} = -(\mathbf{c}_4(1 + c_3l))^{-1} \cdot \left( C_{L_{\alpha}}(l - \phi_{\gamma}(\gamma)) + c_0 + c_1l - c_4\phi_l(l) - \phi_l(l) \right).
\]  
(50)

As far as propulsion is concerned, we inversed \( \eta \) considering linear approximation of the characteristic \( \Phi_i \Phi^\ast_i(\Phi_i) \). The main part of propulsion available is currently used during acceleration of the HSV, and the control allocation problem discussed in Section 5.4 appears slightly during high AoA phases, since thrust remain available even in this situation where drag dramatically increase.

8. CONCLUSION

The global nonlinear control law which has been addressed here is a first attempt to assign a Lyapunov based flight nonlinear control law, allowing to trade with hard nonlinearities like saturations. The assumptions made here are not very restrictive. Extension of such a control law can be performed on any flying vehicle with little small path angle.

There is many perspectives opened by such a nonlinear control law. We have to extend our work to the fully dimensioned model, and to take in account of altitude impact on the dynamics. Others perspectives are to trade with the control allocation problem. We also need to robustify this control law to model uncertainties, using e.g. nonlinear integrators. Thirdly, we have to find guarantees of boundedness of \( \eta \) without obtaining suboptimal gains on \( k_i \). Finally we need to find robust control laws guaranteeing stability under saturated angle of elevators \( \delta \).

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REFERENCES


5 We only shaped \( \gamma \) in order to stabilize altitude.

6 Variation of \( \pm 20\% \) on aerodynamics and propulsion parameters.